Math 257 PDE Assignment 10

[**PDE in polar coordinates**]

1. [**Laplace equation on a half disk**] Express the solution $u = u(r, \theta)$ of the system

$$\Delta u = 0, \quad (0 < r < a,\ 0 < \theta < \pi),$$

$$u(a, \theta) = f(\theta) \quad (0 \le \theta \le \pi), \qquad u_\theta(r, \theta) = 0 \text{ if } \theta = 0 \text{ or } \theta = \pi,$$

as a series of product solutions. Find the formula for the coefficients. Note we have Neumann boundary condition.

2. [**Euler equation**] Let $0 < a < b$. The general solution of the Euler equation

$$r^2 y'' + r y' - m^2 y = 0, \quad (a < r < b),$$

is $y(r) = c_1 r^m + c_2 r^{-m}$. Suppose $y(b) = 0$, Show that the solutions are of the form $y = c[(r/b)^m - (r/b)^{-m}]$. What if we assume $y(a) = 0$ instead?

3. [**Laplace equation in an annulus**] Find the solution $u = u(r, \theta)$ defined in the annulus domain $\Omega = \{(r, \theta) : 1 \le r \le 2, 0 \le \theta \le 2\pi\}$ of the system

$$\Delta u = 0, \quad \text{ in } \Omega,$$

$$u(1, \theta) = 0, \quad u(2, \theta) = \sin 2\theta.$$

The solution in problem 2 will be useful.

4. [**Wave equation on a half disk**] Express the solution $u = u(r, \theta, t)$ of the system

$$u_{tt} = c^2 \Delta u, \quad (0 < r < a,\ 0 < \theta < \pi,\ t > 0),$$

$$u = 0 \quad \text{if } r = a \text{ or } \theta = 0 \text{ or } \theta = \pi,$$

$$u(r, \theta, 0) = f(r, \theta), \quad u_t(r, \theta, 0) = g(r, \theta), \quad (0 < r < a,\ 0 < \theta < \pi),$$

as a series of product solutions. Find the formula for the coefficients. Note we have Dirichlet boundary condition.

5. Repeat problem 4 for heat equation with Neumann boundary condition on x-axis.

* Final Exam time and place: Friday, Dec 9, 3:30pm.