Math 257 PDE   Assignment 2

Due at the beginning of class
on Wednesday, September 21.

[**Power series**]

1. Find the first three nonzero terms of the Maclaurin series of $f(x) = \dfrac{1}{\cos x}$.

   Remark: The Maclaurin series will have only even powers of x (why?) and so can be written in the form
$$\frac{1}{\cos x} = \sum_{n=0}^{\infty} (-1)^n \frac{E_{2n}}{(2n)!} x^{2n}.$$
   The numbers E_{2n} are called the Euler numbers.

2. (a) Find the first three nonzero terms of the Maclaurin series for $f(x) = e^{-x^2} \sin 2x$. The series will have only odd powers of x (why?).

   (b) Determine the radius of convergence of this power series.

[**Power series solutions of ODE**]

In problems 3–6 (resp. 7), use the **power series method** about $x = 0$ (resp. $x = 1$) to find the recurrence relation for the coefficients of the solutions.

3. $y' + e^x y = 0$.
4. $y'' - xy' + y = 0$. Compute the first 4 nonzero coefficients if the initial conditions are $y(0) = 1, y'(0) = 1$.
5. $y' + \dfrac{1}{1+x} y = 0$. Do you recognize the solutions?
6. $y'' - 2y' + y = 0$. Determine the solution if the initial conditions are $y(0) = 1, y'(0) = 1$.
7. $y' + 2xy = 0, y(1) = 1$.

[**Singular points of ODE**]

8. In each part decide if $x = 0$ is an ordinary point or a singular point. In case it is a singular point, determine if it is a regular singular point.

   (a) $y'' + xy' + \dfrac{1}{2 + \sin^2 x}\, y = 0$.

   (b) $(e^x - 1)y'' + y' + \dfrac{1}{x}\, y = 0$.

   (c) $xy'' + 2y' - \dfrac{1}{3x^2}\, y = 0$.