Math 257 PDE Assignment 3

[**More on singular points of ODE**]

1. Find all singular points of the following equation and determine whether each one is regular or irregular:
$$(x - x^3)y'' + (1 - \frac{1}{x})y' = 0.$$

2. Find all real singular points of the following equation and determine whether each one is regular or irregular:
$$\sin(x)y'' + \frac{2}{x}y' + 4y = 0.$$

3. Does $y'' + (1 - \frac{1}{x-5})y' = 0$ with $y(0) = 1, y'(0) = 0$ have a solution of the form $y(x) = \sum a_n(x-1)^n$? Can one calculate $y(-2)$ using this power series? Give a short answer by referencing appropriate theorem(s) in the book.

[**Euler equation**]

For problems 4–5, (a) determine the general solution of the given differential equation that is valid in any interval not including the singular point; (b) Find the solution with the given initial value and describe how the solution behaves as x goes to the singular point.

4. $(x+1)^2y'' + (x+1)y' - y = 0, \quad y(0) = 3, y'(0) = 1.$

5. $x^2y'' + 2xy' + y = 0, \quad y(1) = 0, y'(1) = 1.$

6. $x^2y'' - 3xy' + 4y = 0 \quad y(1) = 1, y'(1) = 0.$

[**Power series solutions near regular singular point**]

For problem 7, write down the recurrence relations for the coefficients of the first and the second series solutions. Determine at least three nonzero terms in each series solution. Take $x > 0$.

7. $2xy'' + (1+x)y' + y = 0.$