Math 257-316 Assignment 4

[**Series solutions near regular singular point; Bessel's equations**]

1. For $xy'' + 3y' + \dfrac{1+x}{x}\, y = 0$, $(x > 0)$, find the value(s) of r so that there exists a solution of the form $\sum_{n=0}^{\infty} a_n x^{n+r}$. Find the recurrence relation for a_n.

2. The equation $\sin(x)y'' = e^x\, y$, $(x > 0)$, has a solution of the form $\sum_{n=0}^{\infty} a_n x^{n+1}$, with $a_0 = 1$. Find a_1 and a_2.

3. *Modified Bessel's equation of order* $p \geq 0$ is

$$x^2 y'' + x\, y' + (-x^2 - p^2)\, y = 0.$$

   This family of equations differ from the usual Bessel equations only on the sign of $x^2 y$. They arise when we deal with PDE in polar coordinates in an annulus $0 < r_1 < r < r_2$.

   (a) find the value(s) of r so that there exists a solution of the form $\sum_{n=0}^{\infty} a_n x^{n+r}$. Find the recurrence relation for a_n.

   (b) For the positive r and $p = 1/2$, solve one solution.

[**Periodic functions**]

4. Determine the minimal (fundamental) period of the given function.

   (a) $\sin 2x$, (b) $\cos \dfrac{x}{2} + 3\sin 2x$, (c) $\dfrac{1}{2+\sin x}$, (d) $e^{\cos x}$.

[**Fourier series**]

5. Find the Fourier series of the $2p$-periodic function $f(x) = \begin{cases} x, & \text{if } 0 \leq x \leq p, \\ 0, & \text{if } -p \leq x \leq 0. \end{cases}$

6. Let $f(x) = 1$ for $-2 \leq x \leq 1$ and $f(x) = x - 2$ for $1 < x < 2$. Extend $f(x)$ to a 4-periodic function.

   (a) Plot the function on the interval $[-6, 6]$.

   (b) Plot its Fourier series (without computing it) on the interval $[-6, 6]$.

- This assignment is due at the beginning of class of Wednesday, October 5.
- The first midterm exam is on Wednesday, October 12.