Math 257-316 Assignment 7

For this assignment, you can use the formulas for general solutions. You don't need to go through the separation of variables. You can use, for $0 \le x \le L$,

$$1 = \sum_{n \text{ is odd}} \frac{4}{n\pi} \sin \frac{n\pi x}{L}, \quad x = \sum_{n=1}^{\infty} \frac{2L(-1)^{n+1}}{n\pi} \sin \frac{n\pi x}{L}, \quad x(L-x) = \sum_{n \text{ is odd}} \frac{8L^2}{(n\pi)^3} \sin \frac{n\pi x}{L}.$$

[**Wave equations**]

1. Find the solution of the initial boundary value problem for $u(x,t)$:

$$\begin{cases} u_{tt} = 4u_{xx}, & (0 < x < 1, t > 0), \\ u(0,t) = 0, \quad u(1,t) = 0, & (t > 0), \\ u(x,0) = f, \quad u_t(x,0) = g, & (0 < x < 1). \end{cases} \tag{1}$$

with

$$f(x) = \sin \pi x + \frac{1}{2} \sin 3\pi x + 3 \sin 7\pi x, \qquad g(x) = \sin 2\pi x.$$

2. Solve Eq. (1) for $u(x,t)$ with $f(x) = x(1-x)$ and $g(x) = 0$.

3. Use d'Alembert's formula to solve Eq. (1) for $u(x,t)$ with $f(x) = \sin \pi x + 3 \sin 2\pi x$ and $g(x) = \sin \pi x$.

4. Let $u(x,t)$ be a solution of the wave equation $u_{tt} = 9u_{xx}$ for $x \in \mathbf{R}$ and $t > 0$, with initial conditions $u(x,0) = \begin{cases} 2, & \text{if } 0 < x < 4 \\ -1, & \text{if } -4 < x < 0 \\ 0, & \text{otherwise} \end{cases}$ and $u_t(x,0) = 0$. Use d'Alembert's formula to plot $u(x,t)$ for $t = 0, 1, 2$.

[**Two dimensional heat and wave equations**]

5. Solve the heat equation $u_t = u_{xx} + u_{yy}$ for x, y in a rectangle $0 < x < 2$ and $0 < y < 1$, with zero boundary condition and initial condition $u(x,y,0) = \sin 3\pi x \sin 3\pi y$.

6. Solve the heat equation $u_t = 4(u_{xx} + u_{yy})$ with zero boundary condition in a unit square ($a = b = 1$) with $u(x,y,0) = f(x,y) = x(1-x)y(1-y)$.

7. Solve the wave equation $u_{tt} = u_{xx} + u_{yy}$ defined for $(x,y,t) \in [0,2] \times [0,2] \times [0,\infty)$ with zero boundary condition and initial conditions $u(x,y,0) = x + y$ and $u_t(x,y,0) = 1$.

Note: Final Exam is scheduled on **Friday December 9, 3:30pm**.