Math 257 PDE Assignment 8
due at the beginning of class
on Wednesday November 9

1. Solve the wave equation $u_{tt} = u_{xx} + u_{yy}$ defined for $(x, y, t) \in [0, 2] \times [0, 2] \times [0, \infty)$ with zero boundary condition and initial conditions $u(x, y, 0) = x + y$ and $u_t(x, y, 0) = 1$.

2. Find the steady state temperature on a 1×1 metal plate if the temperature at one end is kept fixed at T^o and the other 3 ends are kept fixed at 0^o.

3. (a) Suppose $u(x, y)$ satisfies Laplace's equation, $\nabla^2 u = 0$, on the rectangle $0 < x < a,\ 0 < y < b$. Let $v(x, y)$ be the function defined by $v(x, y) = u(x, b - y)$. Show that $v(x, y)$ satisfies Laplace's equation on the rectangle.

   (b) Suppose the solution $u(x, y)$ of the Dirichlet problem

$$\begin{aligned} \nabla^2 u &= 0 \text{ for } 0 < x < 2,\ 0 < y < 1 \\ u(x, 1) &= \text{ some function } f(x) \\ u &= 0 \text{ on the other 3 sides} \end{aligned}$$

   is the function

$$\frac{1}{2}\sin(\pi x)\sinh \pi y - \frac{3}{2}\sin 2\pi x \sinh 2\pi y.$$

   Then what is the solution of the Dirichlet problem

$$\begin{aligned} \nabla^2 u &= 0 \ \text{ for } 0 < x < 2,\ 0 < y < 1 \\ u(x, 0) &= -2f(x) \\ u &= 0 \ \text{ on the other 3 sides ?} \end{aligned}$$

4. (a) Find the solution of the Dirichlet problem

$$\begin{aligned} \nabla^2 u &= 0 \ \text{ for } 0 < x < 1,\ 0 < y < 1 \\ u(x, 0) &= 1,\ u(x, 1) = -1,\ u(0, y) = 1,\ u(1, y) = -1. \end{aligned}$$

   (b) What is the value of $u(1/2, 1/2)$?

5. Solve the Poisson problem

$$\nabla^2 u = 1 \text{ for } 0 < x < a,\ 0 < y < b,\ u = 0 \text{ on the boundary.}$$

6. Find a solution of the Neumann problem

$$\begin{aligned} \nabla^2 u &= 0 \ \text{ for } 0 < x < 1,\ 0 < y < 1 \\ u_x(0, y) &= 0,\ u_x(1, y) = \cos \pi y + \cos 2\pi y \\ u_y(x, 0) &= 0,\ u_y(x, 1) = 0 \end{aligned}$$

Note: the next midterm is Wednesday November 16.