TABLE OF INTEGRALS

1. Elementary integrals

All of these follow immediately from the table of derivatives. They should be memorized.

$$\int \sec x \ dx = \ln|\sec x + \tan x| + C$$

$$\int cf(x)\,dx = c\int f(x)\,dx$$

$$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int c\,dx = cx + C$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \quad \sin x + C$$

$$\int \frac{1}{x^2+1}\,dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

$$\int \tan x \ dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int \cot x \ dx = \ln|\sin x| + C = -\ln|\csc x| + C$$

$$\int \sec x \ dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \ dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

2. A selection of more complicated integrals

These begin with the two basic formulas, change of variables and integration by parts.

$\int f(g(x))g'(x)\,dx = \int f(u)\,du$ where $u = g(x), du = g'(x)dx$ (change of variables)

$\int f(g(x))\,dx = \int f(u)\frac{dx}{du}\,du$ where $u = g(x)$ (different form of the same change of variables)

$$\int e^{cx}\,dx = \frac{1}{c}e^{cx} + C \quad (c \neq 0)$$

$$\int a^x\,dx = \frac{1}{\ln a}a^x + C \text{ (for } a > 0,\ a \neq 1)$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C,\ a \neq 0$$

$$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C,\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\frac{x}{a} + C,\ a > 0$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

To compute $\int \frac{1}{x^2 + bx + c}\,dx$ we complete the square

$$x^2 + bx + c = x^2 + bx + \frac{b^2}{4} + c - \frac{b^2}{4} = \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4}$$

If $c - b^2/4 > 0$, set it equal to a^2; if < 0 equal to $-a^2$; and if $= 0$ forget it. In any event you will arrive after the change of variables $u = x + \frac{b}{2}$ at one of the three integrals

$$\int \frac{1}{u^2 + a^2}\,du, \quad \int \frac{1}{u^2 - a^2}\,du, \quad \int \frac{1}{u^2}\,du$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2 \pm a^2} \pm a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right|\right) + C$$

$\int x^n e^{cx}\,dx = x^n\frac{e^{cx}}{c} - \frac{n}{c}\int x^{n-1}e^{cx}\,dx$, $c \neq 0$. This is to be used repeatedly until you arrive at the case $n = 0$, which you can do easily.