HOMEWORK ASSIGNMENT # 3

due in class on Friday, January 27

Student No: _____ Name (Print): _____

Note: All homework assignments are due in class one week after being assigned. They must be on standard $8\frac{1}{2} \times 11$ size paper and they must be stapled. Assignments which are not stapled will not be accepted. I will not bring a stapler to class. Please enter your student number and name (as it appears on the registrar's list) in the spaces above. SURNAME FIRST IN CAPITALS, and given name second. Please put your answers in the boxes (if provided), show any work in the spaces provided and submit these pages for your assignment.

1. Find the harmonic function $u(x, y)$ on the region $\Omega = \{z \mid y > 0,\ 2 \leq xy \leq 4\}$ that satisfies the boundary conditions $u(x, y) = \begin{cases} \alpha & \text{if } xy = 2 \\ \beta & \text{if } xy = 4 \end{cases}$

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2. Let $f(z) = u(x, y) + iv(x, y)$ be analytic on some domain Ω. Show that the Jacobian of the mapping $\Omega \to \mathbb{R}^2$, $(x, y) \to (u(x, y), v(x, y))$, satisfies $J(x, y) = |f'(z)|^2$.

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3. Suppose $f(z)$ is an entire function satisfying $f(z)$ is real $\forall z$ and $f(0) = 2$. Show that $f(z) = 2\ \forall z \in \mathbb{C}$.

4. (a) Show that the function $u(x, y) = \sin x \cosh y$ is harmonic on $\mathbb{R}^2$.

(b) Find the harmonic conjugate $v(x, y)$ of $u(x, y)$ that satisfies $v(0, 0) = 1$.

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5. Let C be the circle which is the intersection of the plane $ax_1 + bx_2 + cx_3 = d$ with the unit sphere $x_1^2 + x_2^2 + x_3^2 = 1$ (assume a, b, c, d are such that there is a non-trivial intersection). Prove that the stereographic projection of C onto the complex plane $\mathbb{C}$ is either a straight line or a circle.

6. Let $f(z) = 1/z$ be the reciprocal map. Prove that this map corresponds to rotation by π about the x_1-axis under stereographic projection.

7. For each of the following subsets Ω of the unit sphere describe the stereographic projection. Let the positive x_1-axis correspond to longitude 0^o and the positive x_2-axis correspond to longitude 90^o.

   (a) Ω is everything "north of 60", including the 60^{th} parallel.

   (b) Ω is everything "south of 60", not including the 60^{th} parallel.

   (c) Ω is everything between the tropic of cancer ($23^o27'$ north) and the tropic of capricorn ($23^o27'$ south) , but not including these parallels.

   (d) $\Omega = \{(x_1, x_2, x_3) \mid x_1^2 + x_2^2 + x_3^2 = 1, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\}$.

   (e) Ω is the closed portion of the southern hemisphere from longitude 30^o to longitude 60^o.