HOMEWORK ASSIGNMENT # 5

due in class on Friday, March 3

Student No: ____________________ Name (Print): ____

Note: All homework assignments are due in class one week after being assigned. They must be on standard $8\frac{1}{2} \times 11$ size paper and they must be stapled. Assignments which are not stapled will not be accepted. I will not bring a stapler to class. Please enter your student number and name (as it appears on the registrar's list) in the spaces above. SURNAME FIRST IN CAPITALS, and given name second. Please put your answers in the boxes (if provided), show any work in the spaces provided and submit these pages for your assignment.

1. Use Cauchy's Integral Theorem to evaluate the following integrals.

   (a) $\displaystyle\int_C \frac{z}{z^3+1}dz$, where C is the positively oriented circle $|z-2|=2$.

(b) $\displaystyle\int_C \frac{z}{z^2+z-2}dz$, where C is the circle $|z|=3$, oriented in the clockwise direction.

(c) $\displaystyle\int_C \frac{\cos\pi(z-1)}{z(z^2+16)(z^2-16)(z^2+25)^2(z^2-25)^2}dz$, where C is the circle $|z|=\pi$ oriented positively.

2. Let C be a simple closed contour and let D be its interior. Suppose $f(z)$ and $g(z)$ are analytic in D and on C.

   (a) Show that $f(z) = g(z)\ \forall z \in D$ if $f(z) = g(z)\ \forall z \in C$.

   (b) Show that $\displaystyle\int_C \frac{f'(\zeta)}{\zeta - z} d\zeta = \int_C \frac{f(\zeta)}{(\zeta - z)^2} d\zeta\ \forall z \in D$.

   (c) Suppose $f(z)$ is an entire function such that $Im(f(z))$ is bounded. Show that $f(z)$ is a constant.

   (d) Suppose $P(z) = \prod_{j=1}^{j=k} (z - r_j)^{s_j}$ is a polynomial in factored form and C is a positively oriented simple closed contour such that $r_1, \ldots, r_n$ are in the interior of C and the rest of the roots are exterior to C. Show that $\displaystyle\int_C \frac{P'(z)}{P(z)} dz = 2\pi\imath(s_1 + \cdots + s_n)$.

3. Evaluate $\int_C \frac{e^{\imath z}}{(z^2+1)^3} dz$, where C is the circle $x^2 + (y-1)^2 = 1$, oriented positively.