

HOMWORK ASSIGNMENT # 6

due in class on Friday, March 10

Student No: _____ Name (Print): _____

Note: All homework assignments are due in class one week after being assigned. They must be on standard $8\frac{1}{2} \times 11$ size paper and they must be stapled. Assignments which are not stapled will not be accepted. I will not bring a stapler to class. Please enter your student number and name (as it appears on the registrar's list) in the spaces above. SURNAME FIRST IN CAPITALS, and given name second. Please put your answers in the boxes (if provided), show any work in the spaces provided and submit these pages for your assignment.

1. Evaluate the following series:

(a) $\sum_{n=3}^{\infty} (-1)^n \frac{1}{3^n}$.

(b) $\sum_{n=2}^{\infty} \frac{1}{n(n+1)}$.

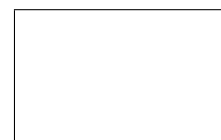
(c) $\sum_{n=1}^{\infty} (-1)^n \frac{z^{2n}}{2^{n+1/2}}$.

2. Determine the radius of convergence of following series.

$$(a) \sum_{n=5}^{\infty} nz^{2n}.$$



$$(b) \sum_{n=0}^{\infty} 4^n z^{3n}.$$



$$(c) \sum_{n=1}^{\infty} \sqrt{n} z^n$$



3. Find closed form expressions for the following series.

(a) $\sum_{n=1}^{\infty} nz^n.$

(b) $\sum_{n=0}^{\infty} (-1)^n \frac{z^n}{(2n+1)!}.$ Hint: the sine function.

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} z^{2n}.$

4. Use the comparison test to show that the following series converge.

$$(a) \sum_{n=1}^{\infty} \frac{\sin(\sqrt{2}n\pi)}{2^n}.$$

$$(b) \sum_{n=1}^{\infty} \frac{n^2 - n - 1}{n^{7/2}}.$$

$$(c) \sum_{n=2}^{\infty} \frac{i^n + (-1)^{n^2}}{n(\sqrt{n} - 1)}.$$

5. Show that the sequence of functions $F_n(z) = \frac{z^n}{z^n - i}$, $n = 1, 2, \dots$ converges to 0 if $|z| < 1$, and to 1 if $|z| > 1$.

6. The Bernoulli numbers B_n are defined by the power series $\frac{z}{e^z - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} z^n$.

(a) Show that $\frac{z}{e^z - 1} + \frac{z}{2} = \frac{z}{2} \coth \frac{z}{2}$.

(b) Show that $B_1 = -1/2$ and $B_3 = B_5 = B_7 = \dots = 0$.

(c) Show that $z \cot(z) = \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n} B_{2n}}{(2n)!} (z)^{2n}$.