HOMEWORK ASSIGNMENT # 7

due in class on Friday, March 30

Student No: ____________________ Name (Print): ____

Note: All homework assignments are due in class one week after being assigned. They must be on standard $8\frac{1}{2} \times 11$ size paper and they must be stapled. Assignments which are not stapled will not be accepted. I will not bring a stapler to class. Please enter your student number and name (as it appears on the registrar's list) in the spaces above. SURNAME FIRST IN CAPITALS, and given name second. Please put your answers in the boxes (if provided), show any work in the spaces provided and submit these pages for your assignment.

1. Determine the nature of all singularities of the following functions $f(z)$.

   (a) $f(z) = \cos 1/z$.

   (b) $f(z) = \dfrac{1}{z^2 \sin z}$.

   (c) $f(z) = \dfrac{z}{e^{z^2} - 1}$.

2. Evaluate the following integrals. In each case the contour is positively oriented.

(a) $\displaystyle\int_{|z|=R} \bar{z}^n dz$, where n is an integer.

(b) $\displaystyle\int_{|z|=3} \cot z dz$.

(c) $\displaystyle\int_{|z-1|=4} \frac{1}{z \sin z} dz$.

3. Let $f(z)$ be the power series $\sum_{n=0}^{\infty} n^2 z^n$.

(a) Find all z such that the power series converges.

(b) Find a closed form expression for $f(z)$.

4. Find all z such that the power series $\sum_{n=1}^{\infty} \frac{1}{n^2} z^n$ converges.

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5. Suppose $f(z)$ is analytic for $|z| \leq 1$ and $|f(z)| \leq M$ for $|z| = 1$, where M is some constant. Show that $|f(0)| \leq M$ and $|f'(0) \leq M$.

6. Determine if there is a function $f(z)$ which is analytic in some open neighbourhood of the origin and which satisfies the following. If there is such a function find a closed form for it and state where $f(z)$ is analytic.

(a) $f^{(k)}(0) = k$ for $k \geq 0$.

(b) $f^{(k)}(0) = (k!)^2$ for $k \geq 0$.

(c) $f(0) = \pi$ and $f^{(k)}(0) = (-1)^{k+1}2^k(k-1)!$ for $k \geq 1$.

7. Evaluate the following integrals. In each case the contour is positively oriented.

(a) $\displaystyle\int_{C_R} \frac{1}{z^2+z+1}\,dz$, where $R > 1$ and C_R is the real axis from $-R$ to R together with the upper half of the circle $|z| = R$.

(b) $\displaystyle\int_{|z|=1} z^2 e^{1/z} \sin(1/z)\,dz$.

8. Evaluate $\int_0^\infty \frac{x^2+1}{x^4+1}dx$.

9. Evaluate $\displaystyle\int_{-\pi}^{\pi} \frac{1}{1+\sin^2\theta} d\theta$.

10. Show that $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^{n+1}}dx = \frac{\pi(2n)!}{2^{2n}(n!)^2}$ for $n = 0, 1, 2, \ldots$