David Brydges, Henri Poincaré Prize Laureate 2024

Laudatio by Gordon Slade, 1 July 2024

Citation: *David Brydges is honored for fundamental results in statistical mechanics and quantum field theory, including the development of rigorous renormalization group methods, the lace and random walk expansions, and deep insights into ϕ^4 field theories.*

Heartfelt congratulations to David Brydges, recipient of a 2024 Henri Poincaré Prize!

David Chandos Brydges was born in Chester, England, son of Michael and Maybery Brydges, on July 1st 1949. Happy 75th birthday David!

His youth was peripatetic, moving from Chester to Hong Kong to London and back to Chester and back to London and then to Birmingham, while attending school in Kent and Surrey. He spent much of a year in Australia before starting at the University of Cambridge in 1967. His subject was Experimental Physics. I don't know if it is true for David, but it is true for at least one of us that an important personal lesson from undergraduate physics lab was that the mathematical side of the subject was the correct future. In any case, David went from Cambridge to the University of Michigan, where he did his PhD in Mathematics during 1970-76 under the supervision of Paul Federbush.

It was in Michigan where his career in mathematical physics was launched, with proofs of Debye screening in various Coulomb systems. This work involved techniques from statistical mechanics and from constructive quantum field theory, including the use of cluster expansions. Statistical mechanics, QFT, and expansion methods have been his intellectual home ever since. Ten papers were published in *J. Math. Phys.* and *Commun. Math. Phys.* during 1974 to 1978 on these topics, with more to come in 1980.

From Ann Arbor, David went to Rockefeller University in New York City to do a postdoc with James Glimm during 1976-1978. He was then at the University of Virginia from 1978 to 2001, where he held the position of Commonwealth Professor. From 2001 until his retirement in 2014, he held the Canada Research Chair in Mathematical Physics at the University of British Columbia.

While at Rockefeller, David got to know Jürg Fröhlich, Erhard Seiler, and Tom Spencer, who became collaborators and friends. It is clear from what

came next that the intellectual sparks were flying between those young mathematical physicists, all thirtyish at the time.

In 1979 and 1980, Brydges, Fröhlich and Seiler published a series of three papers, with a construction of the two-dimensional Abelian Higgs model. These papers continue to be influential today as constructive gauge field theory advances and is taken up by researchers in probability theory.

During 1981 to 1983, David and Jürg Fröhlich published a series of three papers, the first with Tom Spencer and the second and third with Alan Sokal, concerning random-walk representations of classical spin systems and the construction of the ϕ^4_2 and ϕ^4_3 quantum field theories. Devoloping an old idea of Symanzik, they showed how to use random walks to study spin systems and quantum field theory. In the more than forty years since their appearance, these papers have never lost their appeal or their influence.

In 1985, David and Tom Spencer published their paper on a new method, the lace expansion, which they used to prove that weakly self-avoiding walk behaves diffusively in dimensions $d > 4$: the end-point of an n-step walk on average reaches a distance $\sqrt{n}$. This was the start of a major industry that continues to flourish today, including a 2021 paper of David's with Tyler Helmuth and Mark Holmes on the application of the lace expansion to ϕ^4 models with 1- or 2-components. It is typical of David's work that, despite the fact that the lace expansion technique had been around for close to forty years, his paper introduces an original and surprising new expansion and intriguing new insights.

There is not time to talk about all of David's achievements but I do want to mention his beautiful 2003 paper with John Imbrie on branched polymers and dimensional reduction. Inspired by work of Parisi and Sourlas, David and John proved an exact relation between branched polymers in dimension $d + 2$ and the hard-core gas in dimension d. The hard-core gas is easy to solve in dimensions $d = 0$ and $d = 1$, leading to an exact solution to the branched polymer problem in dimensions 2 and 3. To me this paper is an epitome of David's creativity: an idea from the physics literature provides the seed for a mathematical theorem about an object of great interest that is widely considered to be intractable, via a proof that is elegant and suprising enough to be amazing.

David has long been interested in the renomalisation group and its application to critical phenomena. With several different collaboators, he has developed rigorous renormalisation group methods in various directions, at the upper critical dimension and also below the upper critical dimension where the fixed point is non-Gaussian. A fascinating aspect of his work

is that he turned the random walk representation around, by writing the self-avoiding walk as a supersymmetric ϕ^4 model, then using the renormalisation group to analyse the supersymmetric model, and thereby drawing conclusions about the self-avoiding walk. This is a rigorous implementation of de Gennes's statement that the self-avoiding walk is the zero-component ϕ^4 model.

In addition to his many research accomplishments, David has served our profession in several ways, in an editorial capacity for *J. Stat. Phys.*, *Commun. Math. Phys.*, *Probab. Theory Related Fields*, and on Scientific Panels of Institutes (PIMS, Fields, BIRS). He served the IAMP as Treasurer 2000-2003 and as President 2003-2006. His research has been recognised by a Sloan Research Fellowship, a Canada Research Chair in Mathematical Physics, Fellow of the Royal Society of Canada, an Invited Lecture at the International Congress of Mathematicians in 2010, and the Dannie Heineman Prize of the American Physical Society in 2024.

I cannot end my comments without mentioning my personal relationship with David Brydges, as over the past forty years he moved through and combined the roles of teacher, mentor, colleague, collaborator, and friend. It began at the University of British Columbia in 1984, shortly before I defended my PhD. David had come to Vancouver as the External Examiner for my classmate's PhD defence, and while he was there he gave a lecture on the lace expansion. I thought: Wow, that is interesting! Later that summer we were both at a long Summer School in Les Houches, where David's lectures were cancelled because he unfortunately caught chickenpox. It was our first collaboration: I caught shingles (*varicelle et zona en français*—same virus). Fortunately, it went uphill from that point. I went to the University of Virginia as a postdoc where David taught me about the lace expansion and I began to make my own contribution. In 2001 I made my greatest contribution to the University of British Columbia by helping attract David to occupy the Canada Research Chair in Mathematical Physics, where he spent 13 years before retiring and moving to rural Maine. Over the past thirty years we have written 16 joint publications, almost 900 pages, on several topics. The most substantial was the extension of renormalisation group ideas of David going back to 1990, and of isomorphism theorems between supersymmetric spin systems and self-avoiding walk that he developed in the 1980s, that led to an analysis of the critical behaviour of 4-dimensional n-component $|\phi|^4$ models, including the case $n = 0$ which is self-avoiding walk.

During our joint work I have been able to observe David's way of problem

solving. It is the opposite of mine: David starts in the middle, with some very original lemma or idea that is not obviously connected with the end goal, but which turns out to be the key to moving forward. I've also learned to appreciate David as a person, for his kindness and humour and generosity. Like many other colleagues, I've enjoyed the friendship and hospitality of David's family, his wife Betty Lu (herself a prize-winning art quilter) and his daughters Katherine and Suzannah (both accomplished in their scientific fields). A big part of success in scientific research is meeting the right people at the right time: my lucky break was meeting David forty years ago. I have witnessed first hand that many others have greatly profited from their interactions with David, from his warmth and encouragement, and from the generous provision of his insights and good ideas.

Those of you who know David Brydges will have noticed his modesty. It was troubling to him when he was outed as recipient of the 2024 Dannie Hieneman Prize by a fellow mathematician, to his tennis group in rural Maine where he lives, and he told me that for a while it was hard to have a normal conversation. Finally he explained to his tennis friends that "there was not much difference between me and the guy who has a huge model train set in his attic that he sneaks off to whenever he can." He told me "that made sense to both the men and the women and they have forgotten all about it now thank God." So I have to ask you: please do not tell David's tennis group that he has been honoured by the Henri Poincaré Prize. But here and now in Strasbourg, we can celebrate it. Congratulations David Brydges!