Probabilistic Models of Critical Phenomena

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The following formulas are stated in Sections 3.2–3.3:

$$N \sim \begin{cases} 2\epsilon^{-2}\log n & \text{if } \epsilon < 0 \\ An^{2/3} & \text{if } \epsilon = 0 \\ 2\epsilon n & \text{if } \epsilon > 0, \end{cases}$$

the second largest cluster has size $2\epsilon^{-2}\log n$, and

$$\chi(p) \sim \begin{cases} 1/|\epsilon| & \text{if } \epsilon < 0 \\ cn^{1/3} & \text{if } \epsilon = 0 \\ 4\epsilon^2 n & \text{if } \epsilon > 0. \end{cases}$$

The constants $2\epsilon^{-2}, 2\epsilon, 2\epsilon^{-2}, 4\epsilon^2$ in these formulas are not correct when ϵ is fixed. Different constants occur for fixed ϵ. The stated formulas are correct when $\epsilon \to 0$ as $n \to \infty$, but not too quickly, with $|\epsilon|n^{1/3} \to \infty$ (i.e., outside the so-called *critical window*). The formula $1/|\epsilon|$ for $\chi(p)$ when $\epsilon < 0$ is correct both for fixed ϵ and in the above mentioned limit of small ϵ.

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