

Mathematics 305 — Final Exam

Practice Final C

- The test consists of 12 pages and 11 questions worth a total of 60 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.
- No work on this page will be marked.
- If you require extra paper then put up your hand and ask your instructor. You must indicate your name, student number, test number and question number on any extra pages.

Student number								
Section								
Name								
Signature								

1. (a) 4 marks Find all solutions to $\cos z = 2i \sin z$. Write your answer as $z = x + iy$.

- (b) 4 marks Find all solutions to $z^5 + 4 = 0$.

2. 6 marks Construct a branch of $f(z) = (z^3 - 4z)^{1/3}$ such that $f(z)$ is analytic outside the circle $|z| = 2$ and for which $f(x) > 0$ for $2 < x < \infty$. Evaluate $f(2i)$.

3. True or False.

(a) 1 mark $1^z = 1$ for all $z \in \mathbb{C}$.

Answer:

(b) 2 marks $\text{Log}(z^2) = 2 \text{Log}(z)$ for all $z \in \mathbb{C} \setminus \{0\}$.

Answer:

(c) 2 marks $\text{Re}(i/\bar{z}) = -\text{Im}(z)/|z|^2$ for all $z \in \mathbb{C} \setminus \{0\}$.

Answer:

4. 4 marks Evaluate

$$\int_{|z|=2} \frac{\sin z}{z^2(z-4)} dz$$

where the contour runs counterclockwise.

5. 4 marks Evaluate

$$\int_{|z|=2} \frac{e^z}{(z - \pi) \tan(z)} dz$$

where the contour runs counterclockwise.

6. 4 marks Evaluate

$$\int_{|z|=1} z^{-7}(1 - \cos(z))^2 dz$$

where the contour runs counterclockwise.

7. 6 marks Find the Laurent series at $z_0 = 0$ for the function

$$f(z) = \frac{z}{(z-1)(z-2)}$$

which is valid at the point $\frac{3}{2}$. Write your answer as

$$f(z) = \sum_{k=1}^{\infty} a_{-k} z^{-k} + \sum_{k=0}^{\infty} a_k z^k,$$

and find a_{100} and a_{-100} .

8. 5 marks Find the number of zeros of the polynomial

$$f(z) = z^5 + z^4 + 2z^3 - 8z - 1$$

in the right half-plane $\operatorname{Re}(z) > 0$.

9. 6 marks Calculate

$$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 2} dx$$

by complex contour integration.

10. 6 marks Calculate

$$\int_0^\pi \frac{\cos(x-i)}{\sin(x+i)} dx$$

by complex contour integration.

11. 6 marks Calculate

$$\int_0^1 \frac{\sqrt{x(1-x)}}{x+1} dx$$

by complex contour integration. To do this, enclose the slit $[0, 1]$ by a dumbbell contour and enclose the dumbbell contour by a large circle contour.