

# Mathematics 305 — Midterm 1

## Practice Midterm B

- The test consists of 9 pages and 4 questions worth a total of 80 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.

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|----------------|-------|--|--|--|--|--|--|--|
| Student number |       |  |  |  |  |  |  |  |
| Section        |       |  |  |  |  |  |  |  |
| Name           | ..... |  |  |  |  |  |  |  |
| Signature      |       |  |  |  |  |  |  |  |

1. 12 marks (a) 3 marks Find all solutions to  $(z + 1)^6 = z^6$ .

(b) 3 marks Find all solutions to  $\exp(z^2) = -1$ .

(c) 2 marks Calculate the principal value of  $2^{\pi i}$ .

(d) 4 marks Find all solutions to  $\sin(z) = \cos(z)$ .

2. 8 marks

(a) 2 marks Let  $f(x, y) = x$ . Is  $f$  an analytic function?

(b) 2 marks Let  $f(z) = z$  where  $z = x + iy$ . Is  $f$  an analytic function?

- (c) 4 marks Let  $f$  be an analytic branch of  $z^{1/2}$  on  $\mathbb{C} \setminus (-\infty, 0)$  such that  $f(4) = -2$ . Compute  $f(i)$  and write your answer as  $a + ib$  with  $a, b$  real.

3. 10 marks If  $u(x, y) = x^3 - 3xy^2$ , find an entire function  $f$  such that  $\operatorname{Re} f = u$  and  $f(1+i) = -2+i$ . Write your final answer  $f(z)$  as a function of  $z$  (rather than a function of  $(x, y)$ ).

4. 15 marks Find the image  $S' = f(S)$  of the set  $S$  under the map  $f(z)$ .
- (a) 5 marks  $S = \{z : |z| \leq 2 \text{ with } 0 \leq \text{Arg}(z) \leq \pi/4\}$  and  $f(z) = iz^2 + 1$ .

(b) 5 marks  $S = \{z : |z| \leq 1\}$  and  $f(z) = \frac{z+i}{z-i}$ .

- (c) 5 marks  $S = \{z : 0 \leq \operatorname{Re} z \leq 1, \ 0 \leq \operatorname{Im} z \leq \pi/2\}$  and  $f(z) = -e^{2z}$ .