

Mathematics 305 — Midterm 1

Practice Midterm C

- The test consists of 8 pages and 5 questions worth a total of 65 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.

Student number								
Section								
Name								
Signature								

1. 4 marks True or False. For full credit, you must explain your reasoning.

(a) 2 marks $\text{Log}(e^z) = z$ for all $z \in \mathbb{C}$. Here Log is the principal branch of the logarithm.

Answer:

(b) 2 marks $|e^{-z^3}| \leq 1$ for all z with $\text{Re}(z) \geq 0$.

Answer:

2. 13 marks (a) 3 marks Find all solutions to $z^3 = 27i$.

(b) 2 marks Which of the solutions in the previous part is the principal value of $(27i)^{1/3}$?

(c) 4 marks Find all solutions to $e^{1/z} = e^{10}(1 + i)$.

(d) 4 marks Find all solutions to $2 \cos z = i \sin z$.

3. 7 marks Determine where $f(z) = \text{Log}(1 - z^3)$ is analytic. Here $\text{Log}(w)$ denotes the principal branch of the logarithm.

4. 9 marks

(a) 5 marks Find the image $S' = f(S)$ of the set

$$S = \{z : \operatorname{Im}(z) + \operatorname{Re}(z) \geq 1/2\}$$

under the map $f(z) = 1/z$.

- (b) 4 marks Prove that $\Phi(x, y) = \frac{y-1}{x^2+(y-1)^2}$ is a harmonic function when $(x, y) \neq (0, 1)$.

5. 6 marks Find an analytic mapping from $S = \{(x, y) : 1 \leq x \leq 3, y \geq 0\}$ into the upper half plane $\mathbb{H} = \{(u, v) : v \geq 0\}$ so that $(1, 0)$ is mapped to $(-1, 0)$ and $(3, 0)$ is mapped to $(1, 0)$.

If it helps, you may use the identity

$$\sin z = \sin x \cosh y + i \cos x \sinh y.$$