

Mathematics 305 — Midterm 1

Practice Midterm D

- The test consists of 7 pages and 5 questions worth a total of 32 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.

Student number								
Section								
Name								
Signature								

1. 4 marks (a) 4 marks Find all solutions to $\sin z = i$.

(b) 4 marks Find all values of i^{i^i} .

(c) 2 marks Find the principal value of i^{i^i} .

2. 3 marks Write $f(x, y) = x^2 + y^2 + y - 2 + ix$ as a function of $z, \bar{z}$.

3. 3 marks Let $u = xy - x + y$. Find all functions $v(x, y)$ such that $f = u + iv$ is an analytic function.

4. 5 marks Find the image $S' = f(S)$ of the set

$$S = \{z : \operatorname{Re} z > 0\}$$

under the mapping $f(z) = 3(\operatorname{Log} z)$. Here $\operatorname{Log} z$ denotes the principal logarithm.

5. 7 marks Construct a branch of $f(z) = (z^3 - z)^{1/3}$ on $\mathbb{C} \setminus [-1, 1]$ such that $f(2) = \sqrt[3]{6}$. Compute $f(-2)$.