

Mathematics 305 — Midterm 2

Practice Midterm A

- The test consists of 5 pages and 4 questions worth a total of 56 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.
- No work on this page will be marked.
- Fill in the information below before turning to the questions.

Student number								
Section								
Name								
Signature								

1. 10 marks Compute

$$\int_C f(z) dz$$

where C travels along $z = 2e^{i\theta}$ where

$$-\frac{2\pi}{3} \leq \theta \leq \frac{2\pi}{3},$$

and with $f(z)$ given below. Give your answer in the form $a + ib$ with $a, b \in \mathbb{R}$.

(a) 5 marks $f(z) = \frac{1}{z+1}$.

(b) 5 marks $f(z) = \bar{z}^2$.

2. 10 marks Compute $\int_{|z|=2} f(z)dz$, where the circle $|z| = 2$ is oriented counterclockwise and the function $f(z)$ is given below.
- (a) 5 marks $f(z) = (z^2 + 1)e^{1/z}$.
- (b) 5 marks $f(z) = \tan z$.

3. 10 marks Let $a > 0$. Compute

$$\int_0^\infty \frac{\ln x}{x^3 + a^3} dx$$

using complex contour integration.

4. 6 marks Suppose that

$$f : \mathbb{C} \rightarrow \mathbb{C}$$

is analytic. Suppose $f(0) = i$ and $|f(z)| > 1$ for all $|z| = 1$. Prove that there must exist a point z_0 with $|z_0| < 1$ such that $f(z_0) = 0$.