

Mathematics 305 — Midterm 2

Practice Midterm B

- The test consists of 5 pages and 4 questions worth a total of 40 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.
- No work on this page will be marked.
- Fill in the information below before turning to the questions.

Student number								
Section								
Name								
Signature								

1. 10 marks

(a) Compute

$$\int_C \bar{z}^2 \cos z \, dz$$

where C is the circle $|z| = \pi$ running counterclockwise.

(b) Let $f(z)$ be analytic on $|z| \leq 2$ and suppose

$$\max_{|z|=2} |f| \leq 1.$$

Suppose $f(z)$ has a zero of order 3 at $z = 0$. Find the largest possible of $|f(1)|$. Hint: consider $g(z) = f(z)/z^3$.

2. 10 marks

(a) Compute

$$\int_C \frac{1}{z^2(z^4 + 2)} dz$$

where $C = \{|z| = 2\}$ oriented counterclockwise.

(b) Suppose the Taylor series of $f(z) = \frac{1}{1 + e^z}$ at the point $z_0 = i$ is given by the infinite series

$$\sum_{n=0}^{\infty} a_n (z - i)^n.$$

What is the radius of convergence of this series?

3. 10 marks Let $a > 1$. Compute

$$\int_0^{2\pi} \frac{1}{a + \cos \varphi} d\varphi$$

by transforming the integral into a complex contour integral.

4. 10 marks Let

$$f(z) = \frac{z+1}{z^2-3z+2}.$$

Consider the Laurent series of $f(z)$ centered at $z_0 = 0$ that converges for $|z| > 2$. Evaluate the first two terms and compute

$$\int_{|z|=4} f(z) dz$$

where $|z| = 4$ runs counterclockwise.