

Mathematics 305 — Midterm 2

Practice Midterm C

- The test consists of 5 pages and 4 questions worth a total of 40 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.
- No work on this page will be marked.
- Fill in the information below before turning to the questions.

Student number								
Section								
Name								
Signature								

1. 10 marks

(a) Define an analytic function $f : \{\operatorname{Re} z > 0\} \rightarrow \mathbb{C}$ by

$$f(z) = \sum_{n=0}^{\infty} e^{-nz}.$$

Find a closed form expression for $f(z)$ and use it to evaluate

$$\sum_{n=1}^{\infty} ne^{-n}.$$

(b) Suppose we know $\operatorname{Res}(f(z), 0) = a_{-1}$. Let $\alpha \neq 0$ be a complex constant. Evaluate $\operatorname{Res}(f(\alpha z), 0)$.

2. 10 marks Let $C = \{|z| = 1\}$ running counterclockwise. Compute the following integrals.

(a) $\int_C \frac{1}{z(e^z - 1)} dz.$

(b) $\int_C \text{Log}(2 + z) dz$, where Log is the principal branch of the logarithm.

3. 10 marks Calculate the real integral $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 4)}$ using complex integration.

4. 10 marks

- (a) Let $f(z)$ be an analytic function. Suppose $|f(z) - 1| < 1$ for all z with $|z| = 1$. Prove that there is no solution to $f(z) = 0$ with $|z| < 1$.
- (b) Compute $\int_C \sqrt{z} dz$, where $\sqrt{z}$ denotes the principal branch of the square root and C is the straight line from $z_1 = \sqrt{2}$ to $z_2 = -1 + i$.