

Mathematics 305 — Midterm 2

Practice Midterm D

- The test consists of 5 pages and 4 questions worth a total of 38 marks.
- Electronic devices of any kind (including calculators, cell phones, etc.) are forbidden.
- No work on this page will be marked.
- Fill in the information below before turning to the questions.

Student number								
Section								
Name								
Signature								

1. 6 marks Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an analytic function. Suppose $\operatorname{Im} f(z) \geq 0$ for all $z \in \mathbb{C}$. Prove that f is a constant function. To do this, consider

$$g(z) = e^{if(z)}$$

and apply Liouville's theorem.

2. (a) 4 marks Compute

$$\int_C \frac{e^z}{(\sin z)(z-1)^3} dz$$

where C is the circle $|z+2|=1$ running counterclockwise.

- (b) 4 marks Compute

$$\int_C \frac{1}{3i + \sqrt{z}} dz$$

where C is the circle $|z-2|=1$ running counterclockwise.

- (c) 4 marks Compute

$$\int_C \frac{1}{(z-3)(z-5)} dz$$

where C is the circle $|z-2|=2$ running counterclockwise.

3. 10 marks Calculate

$$I = \int_0^{\infty} \frac{\sin x}{x(x^2 + 1)} dx$$

by complex contour integration.

4. 10 marks Find the Laurent series for

$$f(z) = \frac{z^2}{z^2 - z - 2}$$

centered at 0 which converges at $z = \frac{3i}{2}$. Write your answer as

$$a_0 + \sum_{k=1}^{\infty} a_k z^k + \sum_{k=1}^{\infty} a_{-k} z^{-k}.$$

What is a_0 and a_{-1} ?