Differential Topology: Homework 4

Due Date: November 29

- **Problem 1:** Let C be image of

  $$u : \mathbb{CP}^1 \to \mathbb{CP}^4, \quad [X_0, X_1] \mapsto [X_0, -X_0, X_1, -X_1, 0].$$

  Compute the normal bundle of $C \subset \mathbb{CP}^4$.

- **Problem 2:** Show that the Poincaré dual of the ray $\{(x, 0) : x > 0\}$ in $\mathbb{R}^2 \backslash \{0\}$ is $d\theta/2\pi$, where (r, θ) are polar coordinates.

- **Problem 3:**

  (a) Let s be a transverse holomorphic section of $\mathcal{O}(d) \to \mathbb{CP}^2$. Let $\Sigma = s^{-1}(0)$. Compute $\Sigma \cdot \Sigma$.

  (b) Let E be the total space of the bundle $\mathcal{O}(-2) \to \mathbb{CP}^2$. Let $S_0 \subset E$ denote the zero section, and let $C = \{\sum_{i=0}^{2} Z_i^3 = 0\} \subseteq \mathbb{CP}^2$ be a cubic. Compute $S_0 \cdot C$.

- **Problem 4:**

  (a) Compute the cohomology of $U(1)$.

  (b) Compute the cohomology of $U(2)$ by using the fiber bundle structure

  $$U(1) \to U(2) \to S^3.$$

  This fiber bundle structure comes from the action of $U(n)$ on the sphere S^{2n-1} with stabilizer $U(n-1)$. Start by writing down the table for the E_2 page of the spectral sequence.

  (c) Compute the cohomology of $U(3)$, using the fiber bundle structure

  $$U(2) \to U(3) \to S^5.$$

  Continuing like this, one can compute the cohomology of $U(n)$, but there is no need to hand-in the calculation for general n.