TOPICS IN COMPLEX GEOMETRY: PROBLEM SET 2

Due Oct 24

1. The Fubini-Study metric on $\mathbb{P}^n$ is

$$\omega = i\partial\bar{\partial}\log(1+\sum_{i=1}^{n}|z^i|^2), \quad \omega = ig_{j\bar{k}}dz^j \wedge d\bar{z}^k.$$

Show that ω is well-defined on the overlap of two coordinate systems. Compute the matrix $g_{j\bar{k}}$ and show that it is positive definite:

$$g_{j\bar{k}}\xi^j\overline{\xi^k} \geq 0$$

for all $\xi \in \mathbb{C}^n$ and equality if and only if $\xi = 0$. Compute the Chern-Ricci curvature and show

$$R_{j\bar{k}} = (n+1)g_{j\bar{k}}.$$

proving that g is Kähler-Einstein.

2. Let (M, g) be a hermitian manifold, and consider the $\bar{\partial}$-Laplacian $\Delta_{\bar{\partial}}$ on $(1,1)$ forms.

$$(\Delta\beta)_{j\bar{k}} = (\bar{\partial}\bar{\partial}^\dagger\beta)_{j\bar{k}} + (\bar{\partial}^\dagger\bar{\partial}\beta)_{j\bar{k}}$$

Show that Δ is elliptic by computing the symbol via the local expression

$$(\Delta_{\bar{\partial}}\beta)_{j\bar{k}} = -g^{p\bar{q}}\partial_p\partial_{\bar{q}}\beta_{j\bar{k}} + \dots.$$