Course outline

MATH_V 401 Green's Functions and Variational Methods, University of British Columbia
2024 Winter Term 2 (January - April 2025)

Description

In this course we explore two important methods in the study of differential equations. We first study Green's functions of ODE, steady state PDE and time-dependent PDE. We then consider calculus of variations and its applications to the eigenvalue problems.

Prerequisites

Either (a) a score of 80% or higher in one of MATH 256, MATH 257, MATH 316, MATH 358, MECH 358, PHYS 312 or (b) MATH 400.

Topics

1. Green's Functions for linear ODE: (3 lectures)
   - Introduction to generalized functions (distributions)
   - Green's functions for ODEs
   - Sturm-Liouville boundary value problems
2. Green's Functions for steady-state PDE: (5 lectures)
   - Fundamental solutions (Green's functions in $\mathbf{R}^n$)
   - Linear elliptic equations with Dirichlet, Neumann or Robin boundary conditions
   - Green's functions by method of images
   - Green's functions by eigenfunctions in a general domain
   - Layer potentials for boundary value problems
3. Green's Functions for time-dependent PDE: (4 lectures)
   - Fundamental solutions and Duhamel's principle
   - Green's functions by method of images
   - Layer potentials for initial boundary value problems of heat equation
4. Calculus of variations: (6 lectures)
   - Variational problems, brachistochrone
   - Euler-Lagrange equations
   - Constrained variational problems and Lagrange multipliers
   - isoperimetric problem and geodesics
5. Eigenvalue problems: (6 lectures)

- basics
        - Rayleigh quotient and variational principles for eigenvalues
        - eigenvalue bounds
        - Rayleigh-Ritz method
        - Galerkin and finite element methods
6. total: 24 lectures

References

There is no required textbook for this course. I will refer to the following optional books for relevant materials:

- Walter A. Strauss, Partial Differential Equations, An Introduction, John Wiley & Sons, Inc., 2004 2nd ed. I know Prof. Strauss personally and have great respect for him.
- Richard Haberman: Applied Partial Differential Equations, Fourth edition, Pearson Prentice Hall, 2004.
- E. Zauderer: Partial Differential Equations of Applied Mathematics, Wiley. **Its ebook is available in the UBC library**. Here is a link to a **local copy (https://canvas.ubc.ca/ courses/153147/files/37015066?wrap=1)** ⬇ **(https://canvas.ubc.ca/courses/153147/files/37015066/ download?download_frd=1)** .
- I. M. Gelfand and S. V. Fomin, Calculus of variations, Dover.

Important Dates

- First lecture: Tuesday, Jan. 7
- Last day to withdraw without record: Friday, Jan. 17
- midterm break: Feb 17-21
- Midterm exam: Thursday, March 6
- Last lecture: Tuesday, Apr. 8
- Final exam: TBA, during April 12-27

Grading

Homework 16%
  Nine (9) weekly homework due Thursdays Jan 23, 30, Feb 6, 13, 27, Mar 13, 20, 27, and Apr 3 at 11:59pm on Canvas, with the lowest score dropped;
Midterm exam 28%
  One 75-minute midterm exam on Thursday, March 6, in class.
Final exam 56%
  One 150-minute final exam. The final exam is accumulative and covers all topics.

Canvas

Canvas is UBC's mobile-friendly online learning platform. You can log in Canvas with your CWL id. Announcements, assignments, lecture notes, solutions of homework and exam will be all posted in Canvas. You will take photo or scan your assignments and upload them to Canvas.

Piazza

We will have a forum at Piazza (see link on the sidebar, or visit (this Piazza link later). You can ask and answer questions there. It is more efficient than emailing questions to the instructors since many students will have similar questions, the answers from your classmates may be easier to understand, and the process of discussion is also beneficial. Instructor and TA will occasionally check if there are questions unanswered. Following the advice of the math department, the set up is that students can show up as "Anonymous" to classmates, but not to instructors.

Policies on homework and midterms

1. No calculators or notes are allowed in the midterm and final exams.
2. Homework assignments are due 11:59pm at Canvas on Thursdays. Solutions will be posted on Canvas. A selection of the problems will be graded. If you submit homework late, a 25% penalty will be applied for each day late.
3. Permission to shift the weight of your missed midterm exam to the final exam, or to ignore missed assignments, may be granted only in the following circumstances:
   i. prior notice of a valid, documented absence on the scheduled date (e.g. out-of-town varsity athletic commitment with a letter from a coach), or
   ii. notification to the instructor of absence due to a medical condition with a doctor's note. Otherwise, a score of 0 will be given for the missed midterms/assignments. However, the **UBC policy on Academic Concession** **(http://www.calendar.ubc.ca/vancouver/index.cfm?tree=3,329,0,0)** allows students to request academic concession without documentations ONCE per course. For such request please fill the **form** **(https://owncloud.math.ubc.ca/index.php/s/mumsWsljdjR1idJ#pdfviewer)** .
4. The period for final exams is April 12-27, 2025 inclusive. The exact time will be announced by the University in the middle of the term. Students should not make early travel plans that overlap with the scheduled exam period.

Severe Weather Plan

On a lecture day, If there is a severe weather such as big snow that makes it difficult to come to campus, I will send an announcement and move the lecture online by Zoom. The Zoom link will be given in the announcement. If a severe weather happens on the day of the midterm exam, I will send an announcement to postpone the exam to a later lecture time in class.

Statement on UBC's Policies and Resources to Support Student Success

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious and cultural observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available **here (https://senate.ubc.ca/policies-resources-support-student-success)** .

Instructor

1. Tai-Peng Tsai, Math building room 109, phone 604-822-2591, email ttsai@math.ubc.ca , URL https://personal.math.ubc.ca/~ttsai/
2. office hours: TBA, and by appointments.