First Name: $\qquad$ Last Name: $\qquad$
Student-No: $\qquad$ Section:

> Grade:

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## Indefinite Integrals

1. 9 marks Each part is worth 3 marks. Please write your answers in the boxes.
(a) Calculate the indefinite integral $\int \frac{\sin (x)}{\sqrt{\cos (x)}} d x$ for $0<x<\pi / 2$.
Answer:
(b) Calculate the indefinite integral $\int \frac{x+1}{x^{2}+3 x} d x$ for $x>0$.

Answer:

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(c) (A Little Harder): Calculate the indefinite integral $\int x^{2} e^{-x} d x$.

Answer:

## Definite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.
(a) Calculate $\int_{0}^{\pi / 2} \cos ^{3}(x) d x$.
(b) Calculate $\int_{0}^{3} \frac{9 x^{2}}{x^{2}+9} d x$.
Answer:

Answer:
(c) (A Little Harder): Calculate $\int_{1}^{e^{2}} \frac{\ln x}{x^{2}} d x$.

Answer:

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## Riemann Sum, FTC, and Volumes

3. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.
(a) Calculate the infinite sum

$$
\lim _{n \rightarrow \infty} \sum_{i=1}^{n} \frac{4 i^{2}}{n^{3}} \sqrt{1+\frac{i^{3}}{n^{3}}}
$$

by first writing it as a definite integral. Then, evaluate this integral.
Answer:
(b) For $x \geq 0$ define $F(x)$ and $g(x)$ by $F(x)=\int_{0}^{x} \cos ^{2}(t) d t$ and $g(x)=x F\left(x^{2}\right)$. Calculate $g^{\prime}(\sqrt{\pi})$.

Answer:
(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x=-y^{2}$ and $x=-4+y^{2}$ about the vertical line $x=2$. Do not evaluate the integral.

Answer:

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4. (a) 4 marks Write a definite integral with specific limits of integration that determines the finite area enclosed by $y^{2}=x$ and $x=8-2 y$.
(b) 2 marks Evaluate the integral and so compute the area enclosed.

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5. A solid has as its base the region in the $x y$-plane between $y=1-x^{2} / 9$ and the $x$ axis. The cross-sections of the solid perpendicular to the $x$-axis are semi-circles with the diameter of the semi-circle in the base.
(a) 4 marks Write a definite integral that determines the volume of the solid.
(b) 2 marks Evaluate the integral to find the volume of the solid.

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