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Student-No: $\qquad$ Section:

> Grade:

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## Indefinite Integrals

1. 9 marks Each part is worth 3 marks. Please write your answers in the boxes.
(a) Calculate the indefinite integral $\int e^{-2 x} \sqrt{1+2 e^{-2 x}} d x$.
Answer:
(b) Calculate the indefinite integral $\int(x+1) e^{-x} d x$ for $x>0$.

Answer:
(c) (A Littl Harder): Calculate the indefinite integral $\int \tan ^{5}(x) \sec ^{3}(x) d x$.

Answer:

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## Definite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.
(a) Calculate $I=\int_{0}^{\pi / 8} \sin ^{2}(2 x) d x$.
$\square$
(b) Calculate $I=\int_{1}^{e} x^{2} \ln x d x$.
Answer:
(c) (A Little Harder): Calculate $I=\int_{0}^{\infty} e^{-x} \sin (x) d x$. Answer:

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## Riemann Sum, FTC, and Volumes

3. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.
(a) Calculate the infinite sum

$$
\lim _{n \rightarrow \infty} \sum_{i=1}^{n} \frac{2 i}{n^{2}} e^{-4 i^{2} / n^{2}}
$$

by first writing it as a definite integral. Then, evaluate this integral.
Answer:
(b) For $x>0$ define $F(x)=\int_{1}^{x} t^{-1 / 2} d t$ and $g(x)=\sqrt{F\left(x^{2}\right)}$. Calculate $g^{\prime}(2)$.
Answer:
(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x=(y-2)^{2}$ and $x=2-(y-2)^{2}$ about the vertical line $x=-2$. Do not evaluate the integral.

Answer:
4. (a) 4 marks Write a definite integral with specific limits of integration that determines the finite area enclosed by $y^{2}=4-x$ and $x=3 y-6$.
(b) 2 marks Evaluate the integral and so compute the area enclosed.

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5. A solid has as its base the region in the $x y$-plane between $y=1-x^{2} / 36$ and the $x$-axis. The cross-sections of the solid perpendicular to the $x$-axis have a square shape.
(a) 4 marks Write a definite integral that determines the volume of the solid.
(b) 2 marks Evaluate the integral to find the volume of the solid.

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