Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

- The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.

- No memory aids. No calculators. No communication devices or other electronic devices.

- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	Total
Points:	12	8	12	4	8	44
Score:						

This page has been left blank for your workings.

Indefinite Integrals

1. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral $\int (\ln x)^2 \, dx$ for $x > 0$.

Answer:

(b) Calculate the indefinite integral $\int 3x\sqrt{3 - 3x} \, dx$ for $x < 1$.

Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \tan^3(6x) \sec^3(6x)\, dx$.

Answer:

Definite Integrals

2. **8 marks** Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_0^\pi 3\sin^3 x\, dx$.

Answer:

(b) Calculate $\int_1^2 \frac{x-1}{\sqrt{2x+1-x^2}}\, dx$.

Answer:

Riemann Sum and FTC

3. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Which definite integral corresponds to $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i\cos(\frac{i^2}{n^2}+1)}{n^2}$?

(A) $2\int_0^1 x\cos(x^2+1)dx$

(B) $\int_0^2 x\cos(x^2+1)dx$

(C) $\int_0^1 x\cos(x^2+1)dx$

(D) $2\int_0^1 \sqrt{x}\cos(x+1)dx$

(E) $\int_0^2 \sqrt{x}\cos(x+1)dx$

Answer:

(b) Define $F(x)$ and $g(x)$ by $F(x) = \int_x^2 \ln t\, dt$ and $g(x) = x^2\, F(x)$ for $x > 1$. Calculate $g'(1)$.

Answer:

(c) Let $F(x) = \int_{x^2}^{x^3} 6e^{t^2}\, dt$. Find the equation of the tangent line to the graph of $y = F(x)$ at $x = 1$. Tip: recall that the tangent line to the graph of $y = F(x)$ at $x = x_0$ is given by the equation $y = F(x_0) + F'(x_0)(x - x_0)$.

Answer:

Areas and volumes

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. $\boxed{4 \text{ marks}}$ Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = x+5$ and $y = 6\sqrt{x}$ about the vertical line $x = -1$. **Do not evaluate the integral.**

Answer:

$$V = \int_{1}^{25} 2\pi (x+1)\left(6\sqrt{x} - (x+5)\right)\, dx$$

5. (a) 2 marks Sketch by hand the finite area enclosed by $y^2 = x + 3$ and $y = 1 + x$

Answer:

(b) 4 marks Write a definite integral with specific limits of integration that determines this finite area.

Answer:

(c) 2 marks Evaluate the integral to compute the area enclosed.

Answer: