Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

- The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.

- No memory aids. No calculators. No communication devices or other electronic devices.

- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	Total
Points:	12	8	12	4	8	44
Score:						

This page has been left blank for your workings.

Indefinite Integrals

1. $\boxed{\text{12 marks}}$ Each part is worth 4 marks. Please write your answers in the boxes.

    (a) Calculate the indefinite integral $\int x^2 e^{-x} \, dx$ for $x > 0$.

        Answer:

    (b) Calculate the indefinite integral $\int x\sqrt{3-x} \, dx$ for $x < 3$.

        Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \tan^2(8x) \sec^4(8x)\, dx$.

Answer:

Definite Integrals

2. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_0^{2\pi} (xe^{-x^2} + x\cos x)\, dx$.

Answer:

(b) Calculate $\int_3^4 \frac{x-3}{\sqrt{6x-7-x^2}}\, dx$.

Answer:

Riemann Sum and FTC

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Which definite integral corresponds to $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{4i \ln(\frac{2i}{n}+3)}{n^2}$?

    (A) $\int_0^4 x \ln(\frac{x}{2}+3)dx$

    (B) $\int_0^2 (x-3)\ln(x)dx$

    (C) $\int_3^5 (x-3)\ln(x)dx$

    (D) $2\int_0^2 x \ln(x+3)dx$

    (E) $\int_3^5 x \ln(x+3)dx$

Answer:

(b) Define $F(x)$ and $g(x)$ by $F(x) = \int_0^x te^t \, dt$ and $g(x) = 2x\,F(2x+1)$. Calculate $g'(0)$.

Answer:

(c) Let $F(x) = \int_{x^2}^{x^3} 4e^{t^2}\, dt$. Find the equation of the tangent line to the graph of $y = F(x)$ at $x = 1$. Tip: recall that the tangent line to the graph of $y = F(x)$ at $x = x_0$ is given by the equation $y = F(x_0) + F'(x_0)(x - x_0)$.

Answer:

Areas and volumes

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. [4 marks] Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x = y - 5$ and $x = y^2/36$ about the horizontal line $y = -2$. **Do not evaluate the integral.**

Answer:

5. (a) 2 marks Sketch by hand the finite area enclosed by $y^2 + 5 = x$ and $2y = 8 - x$

Answer:

(b) 4 marks Write a definite integral with specific limits of integration that determines this finite area.

Answer:

(c) 2 marks Evaluate the integral to compute the area enclosed.

Answer: