Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

- The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.
- No memory aids. No calculators. No communication devices or other electronic devices.
- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	Total
Points:	12	8	12	4	8	44
Score:						

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Indefinite Integrals

1. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $\int x^2 \sin x \, dx$ for $x > 0$.

   Answer:

   (b) Calculate the indefinite integral $\int 4x\sqrt{3-4x} \, dx$ for $x < 3/4$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \frac{\sqrt{x^2-9}}{x^2}\,dx, x > 3$. Use the following known result: $\int \sec x\,dx = \ln|\sec x + \tan x| + C$. **Write your final answer without any trigonometric function**.

Answer:

Definite Integrals

2. [8 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate $\int_0^{\pi/3} \sec^{3/2} x \, \tan x \, dx$.

   Answer:

   (b) Calculate $\int_{-1}^{0} \frac{x+1}{\sqrt{-2x+1-x^2}} \, dx$.

   Answer:

Riemann Sum and FTC

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Which definite integral corresponds to $\lim_{n\to\infty} \sum_{i=1}^{n} (\frac{i}{n} + 1) e^{-2\frac{i^2}{n^2}} \frac{2}{n}$?

   (A) $2\int_0^1 xe^{-2(x-1)^2} dx$

   (B) $2\int_0^1 (x+1)e^{-2x^2} dx$

   (C) $\int_1^2 xe^{-2(x-1)^2} dx$

   (D) $\int_1^2 (x+1)e^{-2x^2} dx$

   (E) $\int_0^1 (x+1)e^{-2x^2} dx$

   Answer:

   (b) Define $F(x)$ and $g(x)$ by $F(x) = \int_0^x \sin^2 t \, dt$ and $g(x) = x\, F(x^3)$. Calculate $g'(\pi^{1/3})$.

   Answer:

(c) Let $F(x) = \int_{x^2}^{x^3} 3e^{t^2}\, dt$. Find the equation of the tangent line to the graph of $y = F(x)$ at $x = 1$. Tip: recall that the tangent line to the graph of $y = F(x)$ at $x = x_0$ is given by the equation $y = F(x_0) + F'(x_0)(x - x_0)$.

Answer:

Areas and volumes

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. [4 marks] Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = (x-2)^2$ and $y = x + 4$ about the horizontal line $y = 10$. **Do not evaluate the integral.**

Answer:

5. (a) [2 marks] Sketch by hand the finite area enclosed by $y^2 - x = 0$ and $x - 3y = 10$

Answer:

(b) [4 marks] Write a definite integral with specific limits of integration that determines this finite area.

Answer:

(c) [2 marks] Evaluate the integral to compute the area enclosed.

Answer: