Mathematics 101 — Midterm — 45 minutes

14 & 15 February 2019

- The test consists of 10 pages and 5 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is a single question. Question 5 is split into 3 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.

- No memory aids. No calculators. No communication devices or other electronic devices.

- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	Total
Points:	12	8	12	4	8	44
Score:						

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Indefinite Integrals

1. **12 marks** Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $\int \frac{\ln x}{\sqrt{x}} \, dx$ for $x > 0$.

   Answer:

   (b) Calculate the indefinite integral $\int -2x\sqrt{3 + 2x} \, dx$ for $x > -3/2$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \frac{x^2+x+3}{x^3+4x-x^2-4} \, dx$.

Answer:

Definite Integrals

2. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_{-\pi/2}^{\pi/2} 3\cos^3 x \, dx$.

Answer:

(b) Calculate $\int_{-2}^{-1} \frac{x+2}{\sqrt{-4x-2-x^2}} \, dx$.

Answer:

Riemann Sum and FTC

3. **12 marks** Each part is worth 4 marks. Please write your answers in the boxes.

(a) Which definite integral corresponds to $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{\sqrt{i^2 + 9n^2}}{i^2}$?

(A) $\int_0^3 \frac{\sqrt{x^2+1}}{x^2} dx$

(B) $3 \int_0^1 \frac{\sqrt{x^2+1}}{x^2} dx$

(C) $\frac{1}{3} \int_0^1 \frac{\sqrt{x^2+1}}{x^2} dx$

(D) $\int_0^1 \frac{\sqrt{x^2+9}}{x^2} dx$

(E) $\int_0^3 \frac{\sqrt{x^2+9}}{x^2} dx$

Answer:

(b) Define $F(x)$ and $g(x)$ by $F(x) = \int_x^{2x} \cos^2 t \, dt$ and $g(x) = x \, F(x)$. Calculate $g'(\pi)$.

Answer:

(c) Let $F(x) = \int_{x^2}^{x^3} 7e^{t^2}\, dt$. Find the equation of the tangent line to the graph of $y = F(x)$ at $x = 1$. Tip: recall that the tangent line to the graph of $y = F(x)$ at $x = x_0$ is given by the equation $y = F(x_0) + F'(x_0)(x - x_0)$.

Answer:

Areas and volumes

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. $\boxed{\text{4 marks}}$ Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x = -(y-4)^2$ and $x = -2 - y$ about the vertical line $x = 1$. **Do not evaluate the integral.**

Answer:

5. (a) 2 marks Sketch by hand the finite area enclosed by $y^2 = 3 - x$ and $3y = x + 1$

Answer:

(b) 4 marks Write a definite integral with specific limits of integration that determines this finite area.

Answer:

(c) 2 marks Evaluate the integral to compute the area enclosed.

Answer: