

# Mathematics 101 — Midterm — 45 minutes

**13 & 14 February 2020**

- The test consists of 12 pages and 6 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is split into 3 dependent sub-questions. Question 5 is a single question. Question 6 is split into 2 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.
- No memory aids. No calculators. No communication devices or other electronic devices.
- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	6	Total
Points:	8	12	8	8	4	4	44
Score:							

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## Riemann Sum and FTC

1. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the infinite sum

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3i^2}{n^3 \left( \frac{i^3}{n^3} + 2 \right)}$$

by first writing it as a definite integral and then evaluating it.

Answer:

- (b) Define  $F(x)$  and  $g(x)$  by  $F(x) = \int_{2\pi}^x t \sin t \, dt$  and  $g(x) = (x - \sqrt{\pi})F(x^2)$ . Calculate  $g'(\sqrt{\pi})$ .

Answer:

## Indefinite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral  $\int 2(x + 3)^3 \sin((x + 3)^2) dx$ .

Answer:

(b) Calculate the indefinite integral  $\int (5 + 2 \sin \theta)^{\frac{15}{2}} \cos \theta d\theta$ .

Answer:

(c) (A Little Harder): Calculate the indefinite integral  $\int x^3 e^{x^2} dx$ .

Answer:

## Definite Integrals

3. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate  $\int_0^{\pi/4} \sec^4(x) \tan(x) dx$ .

Answer:

(b) Calculate  $\int_0^1 \frac{5x^2}{3x^2 + 3} dx$ .

Answer:

## Areas, volumes and work

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. (a)  Sketch by hand the finite area enclosed between the curves defined by the functions  $y = x^2 + 2$  and  $y + x = 2$

Answer:

- (b)  Write the definite integral with specific limits of integration that determines this finite area.

Answer:

(c) 2 marks Evaluate the integral.

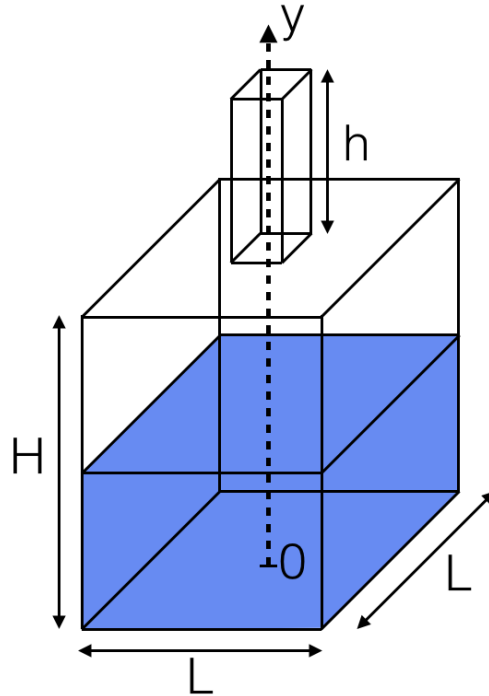
Answer:



5. 4 marks Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between  $y = 3\sqrt{x} + 1$  and  $y = x + 3$  about the vertical line  $x = -2$ . **Do not evaluate the integral.**

Answer:

6. A tank of height  $H$  and of square cross section of edge length  $L$  is half full with water of density  $\rho = 1000\text{kg/m}^3$ . The top of the tank features a spout of height  $h$ . We take the vertical axis  $y$  upwards oriented with its origin at the bottom of the tank. We assume gravity acceleration is  $g = 10\text{m/s}^2$ . We take  $H = 4\text{m}$ ,  $L = 10\text{m}$  and  $h = 2\text{m}$ .



- (a) 2 marks Formulate the total work to pump the water out of the tank by the top of the spout as a definite integral.

Answer:

(b) 2 marks Evaluate the definite integral.

Answer:

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