

Mathematics 101 — Midterm — 45 minutes

13 & 14 February 2020

- The test consists of 12 pages and 6 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is split into 3 dependent sub-questions. Question 5 is a single question. Question 6 is split into 2 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.
- No memory aids. No calculators. No communication devices or other electronic devices.
- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

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| Student number | | | | | | | | |
| Section | | | | | | | | |
| Preferred Name | | | | | | | | |
| Given Name | | | | | | | | |
| Family Name | | | | | | | | |

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|-----------|---|----|---|---|---|---|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| Points: | 8 | 12 | 8 | 8 | 4 | 4 | 44 |
| Score: | | | | | | | |

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Riemann Sum and FTC

1. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the infinite sum

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3i^2 \sin\left(\frac{i^3}{n^3} + 2\right)}{n^3}$$

by first writing it as a definite integral and then evaluating it.

Answer:

- (b) Define $F(x)$ and $g(x)$ by $F(x) = \int_2^x \frac{t}{2t^2+1} dt$ and $g(x) = F(2x) + xF(x)$. Calculate $g'(0)$.

Answer:

Indefinite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral $\int x(x - 2)^5 dx$.

Answer:

(b) Calculate the indefinite integral $\int (5 + 3 \sin \theta)^{\frac{7}{2}} \cos \theta d\theta$.

Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int x (\ln x)^2 dx$ for $x > 0$.

Answer:

Definite Integrals

3. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_{\pi/2}^{\pi} \cos^3(x) \sin^2(x) dx$.

Answer:

(b) Calculate $\int_0^1 \arctan(2x) dx$.

Answer:

Areas, volumes and work

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. (a) Sketch by hand the finite area enclosed between the curves defined by the functions $y + x^2 - 3 = 0$ and $y = -1$

Answer:

- (b) Write the definite integral with specific limits of integration that determines this finite area.

Answer:

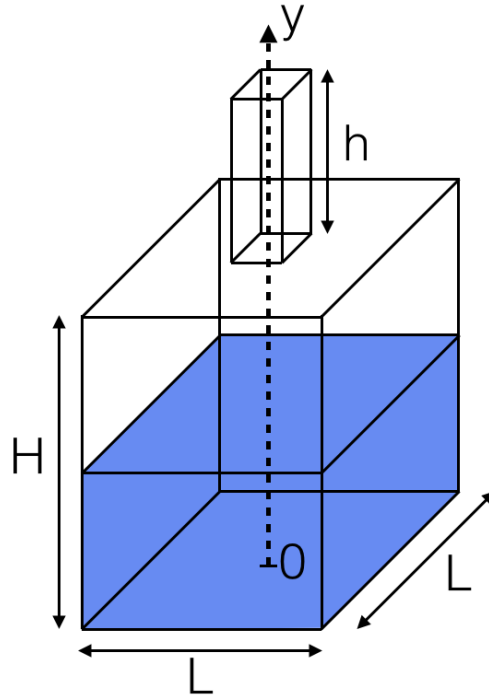
(c) 2 marks Evaluate the integral.

Answer:

5. 4 marks Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x = \frac{(y-1)^2}{9}$ and $x = y - 3$ about the horizontal line $y = 2$. **Do not evaluate the integral.**

Answer:

6. A tank of height H and of square cross section of edge length L is half full with water of density $\rho = 1000\text{kg/m}^3$. The top of the tank features a spout of height h . We take the vertical axis y upwards oriented with its origin at the bottom of the tank. We assume gravity acceleration is $g = 10\text{m/s}^2$. We take $H = 8\text{m}$, $L = 5\text{m}$ and $h = 2\text{m}$.



- (a) 2 marks Formulate the total work to pump the water out of the tank by the top of the spout as a definite integral.

Answer:

(b) 2 marks Evaluate the definite integral.

Answer:

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