

Mathematics 101 — Midterm — 45 minutes

13 & 14 February 2020

- The test consists of 12 pages and 6 questions. Questions 1, 2 and 3 contain multiple independent sub-questions. Question 4 is split into 3 dependent sub-questions. Question 5 is a single question. Question 6 is split into 2 dependent sub-questions. The total number of sub-questions is 13, and is worth a total of 44 marks.
- No memory aids. No calculators. No communication devices or other electronic devices.
- Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work.

Student number								
Section								
Preferred Name								
Given Name								
Family Name								

Question:	1	2	3	4	5	6	Total
Points:	8	12	8	8	4	4	44
Score:							

This page has been left blank for your workings.

Riemann Sum and FTC

1. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the infinite sum

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3i^2 e^{\frac{i^3}{n^3} + 2}}{n^3}$$

by first writing it as a definite integral and then evaluating it.

Answer:

- (b) Define $F(x)$ and $g(x)$ by $F(x) = \int_0^x (2t - 1)e^t dt$ and $g(x) = x^2 F(x)$. Calculate $g'(1/2)$.

Answer:

Indefinite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral $\int (x + 2)(x - 7)^4 dx$.

Answer:

(b) Calculate the indefinite integral $\int (8 + 2 \sin \theta)^{\frac{3}{2}} \cos \theta d\theta$.

Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int e^{-2x} \sin x dx$.

Answer:

Definite Integrals

3. 8 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_0^{\pi/4} \sec^4(x) \tan^3(x) dx$.

Answer:

(b) Calculate $\int_0^1 \frac{7x^2}{5x^2 + 5} dx$.

Answer:

Areas, volumes and work

Please write your answers in the boxes. **Do not use absolute values in your expressions, always work out: (i) the outer function and the inner function for volumes or (ii) which function lies above the other function for areas.**

4. (a) Sketch by hand the finite area enclosed between the curves defined by the functions $y^2 + x = 1$ and $x = y - 1$

Answer:

- (b) Write the definite integral with specific limits of integration that determines this finite area.

Answer:

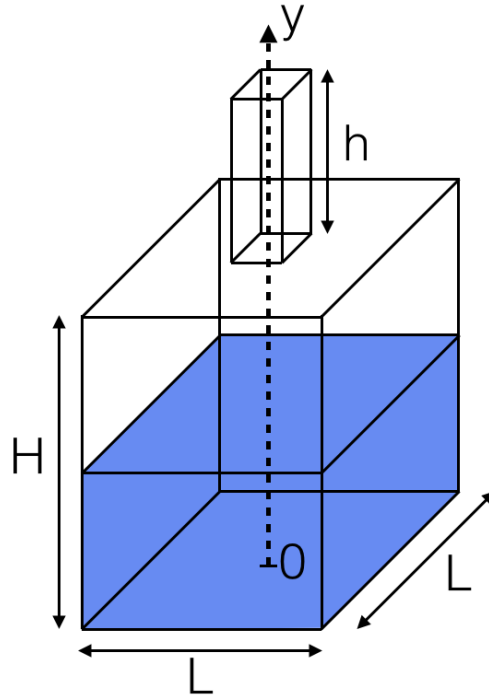
(c) 2 marks Evaluate the integral.

Answer:

5. 4 marks Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x = \frac{(y+1)^2}{25}$ and $x = y - 3$ about the horizontal line $y = -2$. **Do not evaluate the integral.**

Answer:

6. A tank of height H and of square cross section of edge length L is half full with water of density $\rho = 1000\text{kg}/\text{m}^3$. The top of the tank features a spout of height h . We take the vertical axis y upwards oriented with its origin at the bottom of the tank. We assume gravity acceleration is $g = 10\text{m}/\text{s}^2$. We take $H = 8\text{m}$, $L = 2\text{m}$ and $h = 3\text{m}$.



- (a) 2 marks Formulate the total work to pump the water out of the tank by the top of the spout as a definite integral.

Answer:

(b) 2 marks Evaluate the definite integral.

Answer:

This page has been left blank for your workings.