AARMS Course: Homework 2

Problem :

Consider the following problem in the 2-D circular disk $\Omega = \{\mathbf{x} \mid |\mathbf{x}| \leq 2\}$ that contains three small holes

$$\Delta u = 0\,, \qquad \mathbf{x} \in \Omega \backslash \sum_{j=1}^{3} \Omega_{\varepsilon_j}\,,$$
$$u = 4\cos(2\theta)\,, \qquad |\mathbf{x}| = 2\,,$$
$$u = \alpha_j\,, \qquad \mathbf{x} \in \partial\Omega_{\varepsilon_j}\,, \quad j = 1, \ldots, 3\,.$$

(1) Apply the theory for summing infinite logarithmic expansions to first derive a linear system for the source strengths γ_j for $j = 1, \ldots, 3$. (Hint: You will need to know the Green's function for a disk, which can be obtained by the method of images.)

(2) Now assume that each of the three holes has an elliptical shape with semi-axes ϵ and 2ϵ. Assume that the holes are centered at the cartesian coordinate locations $\mathbf{x}_1 = (1/2, 1/2)$, $\mathbf{x}_2 = (1/2, 0)$, and $\mathbf{x}_3 = (-1/4, 0)$. Take the boundary values $\alpha_1 = 1$, $\alpha_2 = 0$, and $\alpha_3 = 2$. By solving the system in (i) numerically, output the source strengths γ_j for $j = 1, .., 3$ versus ϵ for $0 < \epsilon < 0.15$.