

MATH 215 HW #10

IN PROBLEMS 1, 2, 3 BELOW FIND THE GENERAL SOLUTION BY DETERMINING THE PARTICULAR SOLUTION USING THE METHOD OF UNDETERMINED COEFFICIENTS.

PROBLEM 1

$$\underline{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} e^{-2t} \\ -2e^t \end{pmatrix}$$

PROBLEM 2

$$\underline{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t.$$

PROBLEM 3

$$\underline{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} 0 \\ \cos t \end{pmatrix}$$

PROBLEM 4

FIND THE GENERAL SOLUTION TO

$$\underline{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}$$

BY FIRST DIAGONALIZING THE SYSTEM BY CHANGING COORDINATES AS $\underline{x} = S \underline{y}$ FOR SOME APPROPRIATE MATRIX S .

PROBLEM 5

CONSIDER THE LINEAR SYSTEM

$$\underline{x}' = \begin{pmatrix} \alpha & 1 \\ -2 & -3 \end{pmatrix} \underline{x} + \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

WHERE α IS A PARAMETER
($\alpha = \text{CONSTANT}$).

(i) FIND THE PARTICULAR SOLUTION $\underline{x}_p$ FOR ANY α .

(HINT: THERE ARE TWO CASES TO CONSIDER)

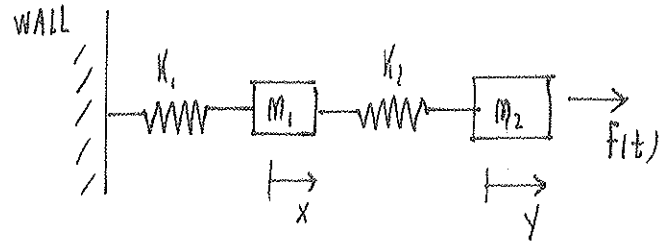
(ii) FIND THE RANGE OF THE PARAMETER α FOR WHICH

$\underline{x}(t) \rightarrow \underline{x}_p$ AS $t \rightarrow \infty$ FOR ANY INITIAL CONDITION $\underline{x}(0)$.

PROBLEM 6 CONSIDER THE COUPLED MASS-SPRING SYSTEM

MODELED BY

$$(*) \quad \begin{cases} m_1 x'' = -K_1 x + K_2 (y - x) \\ m_2 y'' = -K_2 (y - x) + F(t) \end{cases}$$



- (i) IF $m_1 = m_2 = 1$, $K_1 = 5$, AND $K_2 = 6$, AND $F(t) = 0$, FIND THE GENERAL SOLUTION TO $(*)$
- (ii) IF $F(t) = \sin(\omega t)$ FOR WHAT VALUES OF ω (WITH $\omega > 0$) WILL RESONANCE OCCUR?
- (iii) FIND A PARTICULAR SOLUTION FOR $(*)$ WHEN $F(t) = \sin(\omega t)$ IN THE FORM $\underline{x}(t) = \underline{\Gamma} \sin(\omega t)$ FOR SOME $\underline{\Gamma} = \underline{\Gamma}(\omega)$ TO BE FOUND. QUALITATIVELY PLOT $|\underline{\Gamma}(\omega)|$ VERSUS ω . (HERE $|\underline{\Gamma}|$ DENOTES THE LENGTH OF THE VECTOR).