MATH 215 HW # 9

PROBLEM 1 CONSIDER $y'' + 3y' + 2y = g(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$

WITH $y(0) = 0$, $y'(0) = 0$.

(i) CALCULATE $\mathcal{L}(g(t))$.

(ii) solve the differential equation

(iii) calculate $\lim_{t \to \infty} y(t)$.

PROBLEM 2 LET $y(t)$ SOLVE $y'' + 4y = \delta(t-\pi) - \delta(t-2\pi)$

WITH $y(0) = 0$, $y'(0) = 0$.

CALCULATE $y(t)$ USING LAPLACE TRANSFORMS.

PROBLEM 3 LET $\omega > 0$, $T > 0$ AND A BE CONSTANTS, AND CONSIDER THE MASS-SPRING SYSTEM FOR $y(t)$ SUBJECT TO A DELTA-FUNCTION FORCING, MODELED BY

$$y'' + \omega^2 y = A\,\delta(t-T), \quad t \ge 0$$

WITH $y(0) = 1$, $y'(0) = -1$

(i) CALCULATE THE SOLUTION USING LAPLACE TRANSFORMS.

(ii) NOW SET $\omega = 1$. FROM YOUR SOLUTION IN (i) SHOW THAT ONE CAN CHOOSE A AND T SO THAT $y \equiv 0$ FOR ALL $t > T$.

PROBLEM 4 CONSIDER THE THIRD-ORDER DE FOR $y(t)$ GIVEN BY

$$y''' - 4y' = -4 + e^{-t} \quad \text{FOR} \quad t \ge 0$$

WITH $y(0) = 0$, $y'(0) = \alpha$ AND $y''(0) = 0$.

USING LAPLACE TRANSFORMS FIND THE UNIQUE VALUE OF α FOR WHICH

$$\lim_{t \to \infty} y' = 1.$$

(HINT: YOU NEED TO FIND A FORMULA FOR $\mathcal{L}(y''')$.)

PROBLEM 5 USING THE CONVOLUTION FORMULA AND LAPLACE TRANSFORMS

FIND THE SOLUTION TO

$$y'' + w_0^2 y = g(t), \quad \text{WITH} \quad y(0) = y'(0) = 0 \quad \text{AND} \quad w_0 > 0 \quad \text{CONSTANT}$$

FOR AN ARBITRARY $g(t)$.

PROBLEM 6 USE LAPLACE TRANSFORMS AND CONVOLUTION TO SOLVE

$$y'' - 2y' + y = g(t) \quad \text{WITH} \quad y(0) = -1, \; y'(0) = 1$$

AND $g(t)$ IS ARBITRARY.

PROBLEM 7 CONSIDER THE ODE

$$y'' + y = \sum_{n=1}^{\infty} \delta(t - 2n\pi) \quad \text{WITH} \quad y(0) = y'(0) = 0$$

FIND THE SOLUTION $y(t)$ ON THE INTERVAL $2K\pi < t < 2(K+1)\pi$ WHERE $K > 0$ IS AN INTEGER. GIVE A ROUGH PLOT OF y VERSUS t.

PROBLEM 8 SOLVE THE INTEGRAL EQUATION FOR THE UNKNOWN $\phi(t)$ BY TAKING LAPLACE TRANSFORMS AND USING CONVOLUTION.

$$\phi(t) + \int_0^t (t - \xi)\, \phi(\xi)\, d\xi = \sin(2t).$$