MATH 306 HW 4 (25 POINTS)

DO ALL OF THE FOLLOWING PROBLEMS:

PROBLEM 1 (3 POINTS) USING RESIDUE THEORY CALCULATE

$$\int_C \frac{dz}{(z-2)(1+2z^4)(1-3z)^7}$$

WHERE C IS UNIT CIRCLE $|z|=1$ TRAVERSED COUNTERCLOCKWISE.

PROBLEM 2 (3 POINTS): USING RESIDUE THEORY VERIFY $\int_0^1 \sqrt{x(1-x)}\,dx = \frac{\pi}{8}$.

PROBLEM 3 (5 POINTS): USING RESIDUE THEORY VERIFY

$$\int_0^3 \frac{x^{3/4}(3-x)^{1/4}}{5-x}\,dx = \frac{\pi}{2\sqrt{2}}\left(17-40^{3/4}\right).$$

PROBLEM 4 (3 POINTS): SHOW THERE IS EXACTLY ONE ROOT IN THE UNIT DISK $|z|\le 1$ TO $e^z - 4z - 1 = 0$.

PROBLEM 5 (3 POINTS): SHOW THERE ARE NO ZEROES OF

$$z^4 + z^3 + 5z^2 + 2z + 4 = 0$$

IN THE FIRST QUADRANT, RE $z \ge 0$ WITH IM $z \ge 0$.

PROBLEM 6 (3 POINTS): SHOW THAT ALL ROOTS OF $z^8 - 4z^3 + 10 = 0$ ARE IN $1 \le |z| \le 2$.

PROBLEM 7 (5 POINTS): SUPPOSE $p(z) = q_0 + q_1 z + \dots + q_N z^N$ IS A POLYNOMIAL OF DEGREE $N \geq 2$ WITH $q_N \neq 0$ AND $q_0 \neq 0$. SUPPOSE $z_1, \dots, z_N$ ARE DISTINCT ROOTS OF $p(z) = 0$. FOR A SUITABLY CHOSEN CONTOUR C AND BY APPLYING THE RESIDUE THEOREM TO $\int_C \frac{p'(z)}{z^2 p(z)}\, dz$ FIND A FORMULA FOR $S = \sum_{j=1}^{N} \frac{1}{z_j^2}$ IN TERMS OF THE COEFFICIENTS $q_0, q_1, \dots q_N$ OF THE POLYNOMIAL.