

MATH 301: SAMPLE MIDTERM QUESTIONS (M. WARD)

PROBLEM 1: Solve Laplace's equation $T_{xx} + T_{yy} = 0$ in the intersection of the regions $\operatorname{Re}(z) > 1$ and $|z| < 2$. We are given that $T = 1$ on $\operatorname{Re}(z) = 1$ and $T = 3$ on $|z| = 2$.

PROBLEM 2: Find the image of the unit disk $|z| \leq 1$ under the mapping $w = f(z) = \frac{i}{2} \operatorname{Log} \left(\frac{i+z}{i-z} \right)$. (Here Log denotes the principal branch of the logarithm function). (Hint: do it in steps)

PROBLEM 3: Using conformal mapping find the solution $T(x, y)$ to Laplace's equation outside the two circles $C_1 : |z - 2i| = 1$ and $C_2 : |z + 4i| = 2$. You are given that $T = 1$ on C_1 and $T = 2$ on C_2 .

PROBLEM 4: Consider incompressible, inviscid, fluid flow past the ellipse $x^2/a^2 + y^2/b^2 = 1$, where $a > b$. Assume that the flow far from the ellipse has speed V_0 and is inclined at an angle $\alpha > 0$ with respect to the positive x -axis.

- i) Derive the complex velocity potential for the flow
- ii) Derive a formula for the speed of the flow at any point on the ellipse.
- iii) Where are the stagnation points? At what point on the ellipse is the speed maximum?
What is the maximum speed?