HW 0 M316

PROBLEM 1 SOLVE THE FIRST ORDER ODES:

(i) $y' + 3x^2 y = e^{-x^3} \sin x$ WITH $y(0) = 1$.

(ii) $y' = (1 - 2x) y^2$ WITH $y(0) = -1/6$

WHERE IS THE SOLUTION IN (ii) DEFINED?

PROBLEM 2 SOLVE THE FOLLOWING CONSTANT COEFFICIENT SECOND ORDER ODES:

(i) $y'' + 8y' - 9y = 0$ WITH $y(0) = 1$ AND $y'(0) = 0$.

(ii) $y'' - 2y' + 5y = 0$ WITH $y(\pi/2) = 0$, $y'(\pi/2) = 2$.

PROBLEM 3 FOR THE SECOND ORDER LINEAR ODE

$$L(y) \equiv y'' + p y' + q y = 0$$

SUPPOSE THAT y_1 AND y_2 ARE TWO SOLUTIONS. DEFINE THE WRONSKIAN $W(x)$ BY $W = y_1 y_2' - y_1' y_2$. SHOW THAT W SATISFIES

$$W' + pW = 0.$$

PROBLEM 4 VERIFY THAT $y_1 = 1/x$ IS A SOLUTION TO

$$x^2 y'' + 3xy' + y = 0.$$

FIND A SECOND INDEPENDENT SOLUTION USING THE REDUCTION OF ORDER METHOD (I.E. PUT $y = y_1 v$ AND DERIVE/SOLVE THE PROBLEM FOR v).

PROBLEM 5 (RESONANCE)

SOLVE THE INITIAL VALUE PROBLEM FOR $y(t)$:

$$y'' + 4y = 3\sin(2t) \quad \text{WITH} \quad y(0) = 2, \; y'(0) = -1.$$

PROBLEM 6 USING EITHER THE METHOD OF VARIATION OF PARAMETERS, OR THE EXTENDED REDUCTION OF ORDER METHOD, FIND THE GENERAL SOLUTION OF

$$t^2 y'' - 2y = 3t^2 - 1 \qquad \text{FOR } t > 0$$

WHERE $y = y(t)$ AND $y_1 = t^2$ SOLVES THE HOMOGENEOUS PROBLEM.