MATH 316 HW # 2

PROBLEM 1 SOLVE THE INITIAL VALUE PROBLEMS:

(i) $x^2 y'' + 3xy' + 5y = 0$ WITH $y(1) = 1$, $y'(1) = -1$

(ii) $2(x-1)^2 y'' + (x-1)y' - 3y = 0$ WITH $y(2) = 1$, $y'(2) = 4$.

PROBLEM 2 FIND ALL VALUES OF α, WITH α A REAL CONSTANT, FOR WHICH ANY SOLUTION OF $x^2 y'' + \alpha x y' + \frac{5}{2} y = 0$ TENDS TO ZERO AS $x \to 0^+$.

PROBLEM 3 FOR EACH OF THE FOLLOWING FIND ALL SINGULAR POINTS OF THE GIVEN EQUATION AND DETERMINE WHETHER EACH ONE IS REGULAR OR IRREGULAR:

(i) $(x+3)y'' - 2xy' + (1-x^2)y = 0$

(ii) $x^2 y'' - 3\sin x \, y' + (1+x^2)y = 0$

(iii) $x^2(1-x)y'' - (1+x)y' + 2xy = 0$

PROBLEM 4 FOR EACH OF THE PROBLEMS IN PROBLEM 3 THAT HAVE A REGULAR SINGULAR POINT AT SOME $x = x_0$ FIND THE INDICIAL EQUATION AND DETERMINE THE EXPONENTS r IN THE APPROXIMATION $y \cong (x - x_0)^r$.

PROBLEM 5 CONSIDER $2x^2 y'' + 3xy' + (2x^2 - 1)y = 0$.

(i) SHOW THAT $x = 0$ IS A REGULAR SINGULAR POINT

(ii) FIND THE INDICIAL EQUATION AND THE ROOTS OF THE INDICIAL EQUATION.

(iii) SUBSTITUTING $y = \sum_{n=0}^{\infty} a_n x^{n+r}$ FIND THE RECURRENCE RELATION FOR a_n AS WELL AS THE SERIES SOLUTION FOR EACH OF THE ROOTS r IN (ii).

PROBLEM 6 REPEAT PROBLEM 5 FOR

$$x^2 y'' + (6x + x^2) y' + xy = 0.$$

PROBLEM 7 THE LAGUERRE DIFFERENTIAL EQUATION IS

$$xy'' + (1-x) y' + \lambda y = 0 \quad \text{WITH } \lambda \text{ A PARAMETER.}$$

(i) SHOW THAT $x = 0$ IS A REGULAR SINGULAR POINT.

(ii) FIND THE INDICIAL EQUATION, ITS ROOTS, AND THE RECURRENCE RELATION

(iii) FIND ONE SOLUTION FOR $x > 0$. SHOW THAT IF $\lambda = m$ WHERE m IS A POSITIVE INTEGER THEN THIS SOLUTION REDUCES TO A POLYNOMIAL.