MATH 257/316 HW # 7

PROBLEM 1 FIND AN INFINITE SERIES REPRESENTATION FOR THE SOLUTION TO

$$u_t = u_{xx} - xe^{-t}, \quad 0 < x < 1, \; t > 0$$

$$u_x(0,t) = 0, \quad u(1,t) = 1$$

$$u(x,0) = 1$$

(HINT: YOU CAN USE THE INTEGRAL $\int_0^1 x \cos\left[(n+\frac{1}{2})\pi x\right] dx = \frac{(-1)^n}{(n+1/2)\pi} - \frac{1}{(n+1/2)^2\pi^2}$)

PROBLEM 2 THERE IS A GAS LEAK AT THE END $x = \pi$ OF A CORRIDOR $0 < x < \pi$. THE CONCENTRATION OF GAS SATISFIES

$$\begin{cases} u_t = u_{xx} & 0 < x < \pi, \; t > 0 \\ u_x(0,t) = 0, & u_x(\pi, t) = 1 \\ u(x,0) = 0 \end{cases}$$

(i) SHOW FROM THE PDE THAT THERE IS NO STEADY-STATE SOLUTION AND THAT $\int_0^\pi u(x,t)\,dx = t$ FOR ALL $t \geq 0$.

(ii) FIND THE SOLUTION $u(x,t)$ BY FIRST FINDING A PARTICULAR SOLUTION $w(x,t) = ax^2 + bx + ct$ FOR SOME a, b, c WHERE w SATISFIES THE PDE AND THE BC: $w_x(0,t) = 0$, $w_x(\pi, t) = 1$.

THEN WRITE $u(x,t) = w(x,t) + v(x,t)$ AND SOLVE THE HOMOGENEOUS PDE PROBLEM FOR $v(x,t)$ BY EIGENFUNCTION EXPANSIONS.

(HINT: THE INTEGRAL $\int_0^\pi x^2 \cos(nx)\,dx = \frac{2\pi(-1)^n}{n^2}$ FOR $n \geq 1$ MAY BE USEFUL).

PROBLEM 3 SOLVE THE WAVE EQUATION FOR THE DEFLECTION OF A STRING:

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < \pi, \ t > 0$$

$$u(0,t) = u(\pi,t) = 0$$

$$u(x,0) = \sin(2x) + 4\sin(4x), \quad u_t(x,0) = 0.$$

PROBLEM 4 CONSIDER THE DAMPED WAVE EQUATION FOR STRING VIBRATIONS WHERE THE DEFLECTION SATISFIES

$$(*)\quad \begin{cases} u_{tt} + 2u_t = u_{xx}, & 0 < x < 1, \ t > 0 \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = f(x), & u_t(x,0) = g(x) \end{cases}$$

(i) FIND AN INFINITE SERIES REPRESENTATION FOR $u(x,t)$.

(ii) FIND THE EXPLICIT SOLUTION WHEN $f(x) \equiv 0$ AND $g(x) \equiv \sin(4\pi x)$

(iii) FOR THE GENERAL PDE $(*)$, DEFINE THE TOTAL ENERGY $E(t)$ BY

$$E(t) \equiv \frac{1}{2}\int_0^1 (u_x^2 + u_t^2)\, dx.$$

SHOW THAT $dE/dt \leq 0 \quad \forall\, t > 0.$

PROBLEM 5 FIND THE SOLUTION TO THE WAVE EQUATION WITH AN IMPOSED EXTERNAL FORCING MODELED BY

$$u_{tt} = c^2 u_{xx} + \sin(2x)\sin(\omega t) \qquad 0 < x < \pi, \ t > 0$$

$$u(0,t) = u(\pi,t) = 0$$

$$u(x,0) = 0, \quad u_t(x,0) = 3\sin(2x).$$

ALSO, FOR WHAT VALUE OF ω DOES A RESONANCE PHENOMENA OCCUR?

MATH 316/257 HW 7 SOLUTIONS

PROBLEM 1

$$u_t = u_{xx} - xe^{-t}, \quad 0<x<1, \quad t>0$$
$$u_x(0,t) = 0, \quad u(1,t) = 1$$
$$u(x,0) = 1$$

WE FIRST WANT HOMOGENEOUS BC. LET W SATISFY

$$W_{xx} = 0, \quad 0<x<1 \qquad \longrightarrow \qquad W = Ax + B$$
$$W_x = 0 \text{ AT } x=0; \quad W = 1 \text{ AT } x=1 \qquad \text{SO WE GET } W \equiv 1.$$

WE NOW PUT $u = W + V$ WHERE $W = 1$ SO THAT

$$(1) \quad \begin{cases} V_t = V_{xx} - xe^{-t} \\ V_x(0,t) = 0, \quad V(1,t) = 0 \\ V(x,0) = 0. \end{cases}$$

NOW FOR THE PROBLEM WITH NO xe^{-t} TERM WE HAVE THE UNDERLYING EIGENVALUE PROBLEM

$$\Phi'' + \lambda \Phi = 0 \quad \text{IN } 0<x<1$$
$$\Phi'(0) = 0, \quad \Phi(1) = 0$$

WE HAVE FOR $\lambda > 0$ $\quad \Phi(x) = \cos[\sqrt{\lambda}\, x]$ WITH $\Phi(1) = \cos(\sqrt{\lambda}) = 0$

SO $\sqrt{\lambda} = (2n+1)\frac{\pi}{2}$ OR $\lambda_n = (2n+1)^2 \pi^2/4$ WITH $n = 0, 1, 2, \ldots$

THUS $\quad \Phi_n(x) = \cos\left[\frac{(2n+1)\pi x}{2}\right], \quad \lambda_n = (2n+1)^2 \frac{\pi^2}{4}, \quad n = 0,1,2,\ldots$

WE THEN PUT

$$V(x,t) = \sum_{n=0}^{\infty} C_n(t) \cos\left[\frac{(2n+1)\pi x}{2}\right] \qquad (2)$$

WE SUBSTITUTE INTO (1) TO GET

$$\sum_{n=0}^{\infty} C_n' \cos\left[\frac{(2n+1)\pi x}{2}\right] = \sum_{n=0}^{\infty} -C_n \left(\frac{(2n+1)\pi}{2}\right)^2 \cos\left(\frac{(2n+1)\pi x}{2}\right) - xe^{-t}$$

THUS $\sum_{n=0}^{\infty} [c_n' + \lambda_n c_n] \cos\left(\frac{(2n+1)\pi x}{2}\right) = -x e^{-t}$, WHERE $\left.\lambda_n = (2n+1)^2 \frac{\pi^2}{4}.\right\}$ (3)

NOW EXPAND $-xe^{-t} = \sum_{n=0}^{\infty} b_n \cos\left[\frac{(2n+1)\pi x}{2}\right]$

WE OBTAIN $b_n = 2\int_0^1 (-xe^{-t}) \cos\left(\frac{(2n+1)\pi x}{2}\right) dx$

THIS GIVES $b_n = -2e^{-t} \sigma_n$ where $\sigma_n \equiv \int_0^1 x \cos\left[\frac{(2n+1)\pi x}{2}\right] dx$.

WE HAVE THAT $\sigma_n = \frac{(-1)^n}{(n+1/2)\pi} - \frac{1}{(n+1/2)^2\pi^2}$ FROM THE HINT. $\Big\}$ (4)

PUTTING b_n INTO (3) GIVES

$$\sum_{n=0}^{\infty} [c_n' + \lambda_n c_n] \cos\left(\frac{(2n+1)\pi x}{2}\right) = \sum_{n=0}^{\infty} b_n \cos\left(\frac{(2n+1)\pi x}{2}\right)$$

WE CONCLUDE THAT $c_n' + \lambda c_n = b_n$ OR REPLACING $b_n = -2e^{-t}\sigma_n$ WE HAVE

$$c_n' + \lambda_n c_n = -2e^{-t}\sigma_n.$$

NOW SINCE $V(x,0) = 0$ WE HAVE $0 = \sum_{n=0}^{\infty} c_n(0) \cos\left(\frac{(2n+1)\pi x}{2}\right)$, SO THAT $c_n(0) = 0$. THUS, IN SUMMARY,

$$U = 1 + \sum_{n=0}^{\infty} c_n(t) \cos\left(\frac{(2n+1)\pi x}{2}\right)$$ WHERE $c_n(t)$ SATISFIES

$$c_n' + \lambda_n c_n = -2\sigma_n e^{-t}$$
$$c_n(0) = 0.$$
$n = 0, 1, 2, \ldots$ WITH σ_n GIVEN IN (4).

THE SOLUTION IS $c_n(t) = \frac{2\sigma_n}{1-\lambda_n} e^{-t} + Ae^{-\lambda_n t}$ FOR SOME A.

NOW $c_n(0) = 0$ GIVES $A = -2\sigma_n / 1-\lambda_n$.

PROBLEM 2

$$u_t = u_{xx} \quad 0<x<\pi,\ t>0$$
$$u_x(0,t)=0,\quad u_x(\pi,t)=1$$
$$u(x,0)=0$$

(i) THE STEADY-STATE SOLUTION $u_s(x)$, IF IT EXISTS, SATISFIES

$$u_s'' = 0 \longrightarrow u_s = Ax+B.$$
$$u_s'(0)=0,\quad u_s'(\pi)=1$$

NOW $\left.\begin{array}{l} u_s'(0)=0 \to A=0 \\ u_s'(\pi)=1 \to A=1 \end{array}\right\}$ incompatible. HENCE NO steady-state solution.

WE NOW INTEGRATE THE PDE

$$\int_0^\pi u_t\,dx = \int_0^\pi u_{xx}\,dx = u_x\Big|_0^\pi = u_x(\pi,t)=1.$$

THUS $$\frac{d}{dt}\left(\int_0^\pi u(x,t)\,dx\right)=1$$

THIS MEANS $$\int_0^\pi u(x,t)\,dx = t + C.$$

BUT $u(x,0)=0$ YIELDS $\underbrace{\int_0^\pi u(x,0)\,dx}_{=0} = 0 + C$ SO $C=0.$

WE CONCLUDE THAT $$\int_0^\pi u(x,t)\,dx = t.$$

(ii) NOW LET $w = ax^2+bx+ct$ WHERE WE SATISFY

$$w_t = w_{xx}$$
$$w_x(0,t)=0,\quad w_x(\pi,t)=1$$

SO SUBSTITUTING $\quad c = 2a$

$$w_x(0,t)=0 \to b=0$$
$$w_x(\pi,t)=1 \to 2a\pi + b = 1$$

THUS $a=\dfrac{1}{2\pi},\ c=\dfrac{1}{\pi},\ b=0$ SO $w=\dfrac{1}{2\pi}x^2+\dfrac{1}{\pi}t.$

WE NOW PUT $u(x,t) = w(x,t) + v(x,t)$.

WE HAVE $w_t + v_t = w_{xx} + v_{xx}$ BUT $w_t = w_{xx}$

SO $v_t = v_{xx}$

AND
$$u_x(0,t) = w_x(0,t) + v_x(0,t)$$
$$0 = 0 + v_x(0,t) \longrightarrow v_x(0,t) = 0$$
$$u_x(\pi,t) = w_x(\pi,t) + v_x(\pi,t)$$
$$1 = 1 + v_x(\pi,t) \longrightarrow v_x(\pi,t) = 0.$$

$$u(x,0) = w(x,0) + v(x,0)$$
$$0 = \frac{1}{2\pi}x^2 + v(x,0) \longrightarrow v(x,0) = -\frac{1}{2\pi}x^2.$$

WE CONCLUDE THAT
$$v_t = v_{xx} \quad 0<x<\pi,\ t>0$$
$$v_x(0,t) = 0,\quad v_x(\pi,t) = 0$$
$$v(x,0) = -\frac{1}{2\pi}x^2.$$

SEPARATING VARIABLES $v = \Phi T$ SO $\frac{T'}{T} = \frac{\Phi''}{\Phi} = -\lambda \longrightarrow \begin{array}{l}\Phi'' + \lambda\Phi = 0,\ 0<x<\pi \\ \Phi'(0) = \Phi'(\pi) = 0.\end{array}$

WE CONCLUDE THAT $\lambda_0 = 0$, $\Phi_0 = 1$ AND $\lambda_n = n^2$, $\Phi_n(x) = \cos(nx)$.

THEN $T_0 = 1$, $T_n = e^{-n^2t}$. WE OBTAIN THAT
$$v(x,t) = c_0 + \sum_{n=1}^{\infty} c_n e^{-n^2t}\cos(nx)$$
WITH $c_0 = \frac{1}{\pi}\int_0^\pi v(x,0)\,dx$

$c_n = \frac{2}{\pi}\int_0^\pi v(x,0)\cos(nx)\,dx$

THIS GIVES, $c_0 = -\frac{1}{2\pi^2}\int_0^\pi x^2\,dx = -\frac{1}{2\pi^2}\left(\pi^3/3\right) = -\frac{\pi}{6}$.

NOW $c_n = -\frac{1}{\pi^2}\int_0^\pi x^2\cos(nx)\,dx = -\frac{1}{\pi^2}\left[\frac{2\pi(-1)^n}{n^2}\right] = +\frac{2(-1)^{n+1}}{\pi n^2}$ FOR $n = 1, 2, \ldots$

WE HAVE
$$u(x,t) = \frac{1}{2\pi}x^2 + \frac{t}{\pi} - \frac{\pi}{6} + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{\pi n^2}e^{-n^2t}\cos(nx).$$

PROBLEM 3

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < \pi, \ t > 0$$
$$u(0,t) = u(\pi,t) = 0$$
$$u(x,0) = \sin(2x) + 4\sin(4x), \quad u_t(x,0) = 0.$$

SOLUTION SEPARATION OF VARIABLES: $u = \Phi(x)T(t)$ SO

$$\frac{T''}{c^2 T} = \frac{\Phi''}{\Phi} = -\lambda \qquad \begin{array}{l}\Phi'' + \lambda\Phi = 0 \\ \Phi(0) = \Phi(\pi) = 0\end{array} \qquad \begin{array}{l}\Phi_n(x) = \sin(nx) \\ \lambda_n = n^2.\end{array}$$

THEN, $T_n'' + \lambda c^2 T_n = 0$ OR $T_n'' + (nc)^2 T_n = 0 \rightarrow T = A_n\cos(nct) + B_n\sin(nct)$

WE HAVE $$u(x,t) = \sum_{n=1}^{\infty}\left[A_n\cos(nct) + B_n\sin(nct)\right]\sin(nx)$$

NOW $$u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx) = \sin(2x) + 4\sin(4x)$$

WE CONCLUDE THAT $A_2 = 1$, $A_4 = 4$ WITH $A_1 = A_3 = A_5 = A_6 = \ldots = 0$

NOW $$u_t(x,0) = 0 = \sum_{n=1}^{\infty} -ncB_n\sin(nx) = 0 \quad \text{SO} \quad B_n = 0 \quad \forall n = 1,2,3,\ldots$$

THUS $$u(x,t) = \cos(2ct)\sin(2x) + 4\cos(4ct)\sin(4x).$$

PROBLEM 4

$$u_{tt} + 2u_t = u_{xx}, \quad 0 < x < 1, \ t > 0$$
$$u(0,t) = u(1,t) = 0$$
$$u(x,0) = f(x), \quad u_t(x,0) = g(x).$$

(i) WE SEPARATE VARIABLES: $u = T(t)\Phi(x)$ SO $T''\Phi + 2T'\Phi = T\Phi''$

THIS GIVES $$\frac{T''}{T} + \frac{2T'}{T} = \frac{\Phi''}{\Phi} = -\lambda$$

SO $$\begin{array}{l}\Phi'' + \lambda\Phi = 0 \\ \Phi(0) = \Phi(1) = 0\end{array} \longrightarrow \Phi_n(x) = \sin(n\pi x), \quad \lambda_n = n^2\pi^2, \quad n = 1,2,3\ldots$$

THEN $$T_n'' + 2T_n' + \lambda_n T_n = 0.$$

WE PUT $T_n = e^{rt}$ so $r^2 + 2r + \lambda_n = 0$ $\quad r^2+2r+1 = 1-\lambda_n = 1-n^2\pi^2 < 0$

$$(r+1)^2 = 1-n^2\pi^2 \quad \text{so} \quad r = -1 \pm i\sqrt{n^2\pi^2-1}, \quad n=1,2,\ldots$$

WE CONCLUDE THAT $T_n = e^{-t}\left[A_n \cos(\omega_n t) + B_n \sin(\omega_n t)\right]$, $\omega_n \equiv \sqrt{n^2\pi^2-1}$

THUS, $$u(x,t) = \sum_{n=1}^{\infty} e^{-t}\left[A_n \cos(\omega_n t) + B_n \sin(\omega_n t)\right] \sin(n\pi x). \qquad (0)$$

NOW WITH $u(x,0) = f(x) = \sum_{n=1}^{\infty} A_n \sin(n\pi x)$ $\qquad A_n = 2\int_0^1 f(x)\sin(n\pi x)\,dx \quad (1)$

$$u_t(x,t) = \sum_{n=1}^{\infty} -e^{-t}\left(A_n \cos(\omega_n t) + B_n \sin(\omega_n t)\right)\sin(n\pi x)$$
$$+ \sum_{n=1}^{\infty} e^{-t}\left[-A_n\omega_n \sin(\omega_n t) + B_n\omega_n \cos(\omega_n t)\right]\sin(n\pi x)$$

WE PUT $$u_t(x,0) = \sum_{n=1}^{\infty}\left[-A_n + B_n\omega_n\right]\sin(n\pi x) = g(x)$$

NOW $$-A_n + \omega_n B_n = 2\int_0^1 g(x)\sin(n\pi x)\,dx$$

NOW $$B_n = \frac{1}{\omega_n}\left[2\int_0^1 f(x)\sin(n\pi x)\,dx + 2\int_0^1 g(x)\sin(n\pi x)\,dx\right]. \qquad (2)$$

NOW THIS GIVES A_n, B_n FROM (1) AND (2) AND $u(x,t)$ given IN (0)

(ii) NOW IF $g(x) = \sin(4\pi x)$, $f(x) \equiv 0$ THEN $A_n \equiv 0$.

$$\sum_{n=1}^{\infty} B_n\omega_n \sin(n\pi x) = \sin(4\pi x) \qquad B_4 = \frac{1}{\omega_4}, \quad B_n = 0 \text{ FOR } n \neq 4.$$

THUS $$u(x,t) = \frac{e^{-t}}{\omega_4}\sin(\omega_4 t)\sin(4\pi x) \quad \text{WITH } \omega_4 = \sqrt{16\pi^2-1}.$$

(iii) $$U_{tt} + 2U_t = U_{xx}$$

WE DEFINE $$E(t) = \frac{1}{2}\int_0^1 (U_t^2 + U_x^2)\,dx.$$

integrate by parts $u = U_x \quad du = U_{xx}\,dx$

$dv = U_{xt}\,dx \quad v = U_t$

$$dE/dt = \int_0^1 (U_t U_{tt} + U_x U_{xt})\,dx$$

$$= \int_0^1 (U_t U_{tt} - U_t U_{xx})\,dx + U_x U_t \Big|_0^1.$$

BUT $U(0,t) = U(1,t) = 0 \quad \forall t$ SO $U_t(0,t) = U_t(1,t) = 0.$

THIS MEANS $dE/dt = \int_0^1 U_t(U_{tt} - U_{xx})\,dx$ BUT $U_{tt} - U_{xx} = -2U_t.$

WE CONCLUDE THAT $$dE/dt = -2\int_0^1 (U_t)^2\,dx \leq 0.$$

THIS MEANS THAT DUE TO THE DAMPING TERM THE TOTAL ENERGY IS DECREASING.

<u>PROBLEM 5</u> $$U_{tt} = c^2 U_{xx} + \sin(2x)\sin(\omega t)$$

$$U(0,t) = U(\pi,t) = 0$$

WITH $U(x,0) = 0, \quad U_t(x,0) = 3\sin(2x).$

<u>SOLUTION</u> THE EIGENFUNCTIONS ASSOCIATED WITH THE UNFORCED PROBLEM WHERE WE NEGLECT $\sin(2x)\sin(\omega t)$ IS $\Phi_n(x) = \sin(nx).$

AS SUCH WE EXPAND $$U(x,t) = \sum_{n=1}^{\infty} b_n(t)\sin(nx). \qquad (1)$$

WE SUBSTITUTE INTO THE PDE $$\sum_{n=1}^{\infty} b_n''\sin(nx) = \sum_{n=1}^{\infty} -c^2n^2 b_n \sin(nx) + \sin(2x)\sin(\omega t)$$

WE HAVE $$\sum_{n=1}^{\infty} [b_n'' + n^2c^2 b_n]\sin(nx) = \sin(2x)\sin(\omega t).$$

WE CONCLUDE THAT

$$b_n'' + w_n^2 \, b_n = \begin{cases} \sin(wt) & \text{if } n=2 \\ 0 & \text{if } n \neq 2 \end{cases} \qquad \text{WHERE} \quad w_n = nc.$$

NOW $\quad u(x,0) = \sum_{n=1}^{\infty} b_n(0)\sin(nx) = 0 \quad \rightarrow \quad b_n(0) = 0 \quad \forall n = 1,2,\dots$

$$u_t(x,0) = 3\sin(2x) = \sum_{n=1}^{\infty} b_n'(0)\sin(nx) \quad \rightarrow \quad b_2'(0) = 3, \quad b_n'(0) = 0 \quad \forall n \neq 2.$$

WE CONCLUDE THAT

$$(2) \begin{cases} b_2'' + w_2^2 \, b_2 = \sin(wt) & w_2 = 2c \\ b_2(0) = 0, \quad b_2'(0) = 3 \end{cases}$$

$$\begin{cases} b_n'' + w_n^2 \, b_n = 0 \quad \text{FOR } n \neq 2 \\ b_n(0) = b_n'(0) = 0 \end{cases} \Longrightarrow \quad b_n \equiv 0 \quad \forall t \text{ if } n \neq 2.$$

SO $\quad u(x,t) = b_2(t)\sin(2x) \quad$ WHERE $\; b_2(t) \;$ SATISFIES $\;(2).$

NOTICE THAT IF $\; w = w_2 \;$ THEN WE HAVE RESONANCE AND $b_2(t)$ IS NOT BOUNDED AS $t \to \infty$.

- SUPPOSE $\; w \neq w_2 \;$ (NON-RESONANCE). THEN THE PARTICULAR SOLUTION b_{2p} IS $\; b_{2p} = A\sin(wt) \;$ SO $\; -Aw^2 + Aw_2^2 = 1 \;$ SO $\; A = \dfrac{1}{w_2^2 - w^2}.$

  THUS $\quad b_2 = A\cos(w_2 t) + B\sin(w_2 t) + \dfrac{1}{w_2^2 - w^2}\sin(wt)$

  NOW $\; b_2(0) = 0 \rightarrow A = 0.$ NOW $\; b_2'(0) = 3 = Bw_2 + \dfrac{w}{w_2^2 - w^2} \;$ SO $\; B = \dfrac{1}{w_2}\left(3 - \dfrac{w}{w_2^2 - w^2}\right)$

  THUS $\quad b_2 = \dfrac{1}{w_2}\left(3 - \dfrac{w}{w_2^2 - w^2}\right)\sin(w_2 t) + \dfrac{1}{w_2^2 - w^2}\sin(wt) \quad$ IF $\; w \neq w_2$

  AND $\quad u(x,t) = b_2(t)\sin(2x).$

- NOW SUPPOSE $\omega = \omega_2$ RESONANT CASE.

NOW PUT $\hat{b}_2'' + \omega_2^2 \hat{b}_2 = e^{i\omega_2 t}$

SO $\hat{b}_2 = A t e^{i\omega_2 t}$ SO $\hat{b}_2' = A e^{i\omega_2 t} + A t \, i\omega_2 e^{i\omega_2 t}$

AND $\hat{b}_2'' = 2 i A \omega_2 e^{i\omega_2 t} - A t \omega_2^2 e^{i\omega_2 t}$. SUBSTITUTING,

$$2 i A \omega_2 - A t \omega_2^2 + A t \omega_2^2 = 1 \quad \text{SO} \quad A = \frac{1}{2 i \omega_2} = \frac{-i}{2\omega_2}.$$

THUS $b_{2p} = \mathrm{IM}\left[\frac{-i t}{2\omega_2} e^{i\omega_2 t} \right] = -\frac{t}{2\omega_2} \cos(\omega_2 t)$.

THUS $b_2 = A \cos(\omega_2 t) + B \sin(\omega_2 t) - \frac{t}{2\omega_2} \cos(\omega_2 t)$

NOW $b_2(0) = 0 \rightarrow A = 0$

$b_2'(0) = 3 \rightarrow B\omega_2 - \frac{1}{2\omega_2} = 3$ SO $B = \frac{1}{2\omega_2^2} + \frac{3}{\omega_2}$.

THUS $b_2(t) = \left(\frac{1}{2\omega_2^2} + \frac{3}{\omega_2} \right) \sin(\omega_2 t) - \frac{t}{2\omega_2} \cos(\omega_2 t)$

AND $u(x,t) = b_2(t) \sin(2x)$.