MATH 345: LAB 1, THE PHASE LINE

Purpose: The purpose of this lab is to use the ODE software package **xppaut** to expore the dynamics of a simple logistic growth model with harvesting. The package **xppaut** is very large and complicated and has many options. For the first lab we will play with only a VERY SMALL number of commands in **xppaut**.

The Model: The mathematical model of logistic growth subject to **constant harvesting** can be cast in the dimensionless form

$$\frac{dx}{dt} = x(1-x) - \gamma\,, \qquad x(0) = x_0$$

Here the carrying capacity is one and γ denotes the dimensionless harvesting term.

After logging into the Windows system (in LSK; see lab instructions file) or if you have the software on your own machine, please copy and paste into your main directory on inferno the following two files:

harvest.ode and harvestper.ode

These two files are given on my webpage on the line of Lab 1.

Loading the ODE system: In particular, the harvest.ode file has an explicit form that is readbable by xppaut:

```
# harvesting ode
dx/dt = x*(1-x) -gam
param gam=0.0
@ total=10, xhi=10, xlo=0.0, yhi=1.5, ylo=0.0
done
```

The first line starting with # indicates a comment line. The second line specifies the ode. The third line indicates that gam is a paramater whose value is set to 0 (this can be changed later at the terminal). The fourth line indicates that the final time to integrate to, denoted by total, is 10, and that the horizonatal axis (the time axis) runs from 0.0 to 10.0 while the vertical axis (the population axis) runs from 0.0 to 1.5. This will set the boundary of the initial box when **xppaut** is called.

Calling the Program: To invoke the program follow the steps indicated

1. In your main directory, type *xppaut* and hit *return*. (this calls the executable)
2. You will then get a window that appears where you can click on the file harvest.ode that you wish to load.

3. At this stage XPPAUT opens a big window with MANY options should appear. We will examine only a few of the large number of options in this lab.

There are many things that one can do now including; computing trajectories for specific initial conditions, saving these trajectories in order to obtain a hardcopy plot, putting titles and labels onto the figure, and repeating the above by changing the value of the paramater gamma directly at the terminal. **The left mouse button will be the main tool. Clicking is done with the left mouse button**.

1. SOME OF THE COMMANDS IN XPPAUT

Computing Trajectories: To specify an initial condition place the mouse over the *Initialconds* option and click with the left mouse button. Another set of options should appear. Drag the mouse under the *new* heading and click with the left mouse button. Then use the delete key to erase the 0 and type the value 1.45 into the *command line* and hit *return.* A trajectory should be drawn in the black area starting from the specified initial point $x(0) = 1.45$. To save this curve for a hardcopy plot click the left mouse button on the *Graphic stuff* option. Drag the mouse under the *Freeze* option and click with the left mouse button. Another menu then appears. Choose the *Freeze* option and then click a final time with the left mouse button. A little window should pop up and you are to click with the left mouse button on the *OK* symbol. This will save this trajectory when you plot later. **Important:** Every time you want to save a trajectory for plotting these steps involving the *Freeze* commands are needed. A failure to do so will result in the trajectory not being permanantly recorded.

Now one can repeat this procedure by choosing different initial conditions such as 1.4, 1.3, 1.2, 1.0, 0.8, 0.6, 0.4, and 0.2. Compute and freeze the trajectories for each one of these initial conditions. The resulting picture will have many trajectories on it. If the trajectory goes out of bounds you will be notified and simply click with the left mouse button on the *OK* symbol.

Labelling the Axes: To insert labels into the picture click the left mouse button onto the *Text, etc* option. Choose the *Text X* option with the left mouse button. This places you at the command line. Type *Gamma=0* at the command line and hit *return* three times. A little window which says *"Place text with Mouse"* appears. Position the arrow within the border of the back box and click the left mouse button where you would like the label to appear. This label then indicates the parameter value of gamma corresponding to this plot. Repeat these steps to put convenient labels onto the horizontal and vertical axes.

Saving the Picture as a PostScript file: Now we are done with this plot and we would like to save it to obtain a hardcopy version of it later. To do this click on *Graphic stuff* and then click on the *Postscript* submenu (clicking is always done with the left mouse button). This places you at the command line and gives a suggested name for the .ps file which will be saved. I suggest using the delete key to go backwards and name this file *harvgam0.ode.ps* to indicate that it was done with gamma=0. This will put the file in your own directory (not in xpp, which is writeable only by me). Hit return twice, which should save the picture as a .ps file. **NOTE: be careful about what name you choose since in saving the file it will OVERWRITE any existing file in the odes/ subdirectory that has the same name**.

Changing Parameters: We now would like a new set of trajectories for the value $\gamma = .10$. To do so we first need to erase the buffer containing the frozen old trajectories. To do so choose the *Graphic stuff* option, click on *Freeze* and the click on *Remove all.* Then click on *Erase.* This will eliminate the old trajectories. Now go into the *Text* option and either click on *Delete all* or *edit* to either delete all the labels or only certain selected labels that will appear as prompts. Now to change γ at the terminal, first position the mouse under the *Par* icon in the lower left corner of the screen (it is the 5th icon from the left). Double click on the icon (two rapid clicks with the left mouse button) will open the icon. Move the little window out of the way. Select the γ parameter by clicking on the thin rectangle containing $\gamma = 0$. The command line is then prompted. Give a value of .10 for γ on this line and hit *return*. Notice the little window now gives $\gamma = .10$ for the value of γ. Click on the *done* symbol in the window. Now to close the icon hold the left mouse button down over the top left corner of the window. A little menu should appear. Drag the mouse under the *minimize* option and release the left mouse button. **Do not click on CLOSE as this will bring the session with xppaut to an ungraceful end**. The little window should disappear and the *Par* icon should return to its place at the bottom of the screen. The parameter γ is then kept at .10 and you can then plot different trajectories.

To Quit: Click on the *File* command and then click on the *quit* option in the submenu. Click on *yes* in the little window asking you whether you are sure. The windows should dissappear and you will be returned to the unix prompt line. To view your .ps file at the terminal type *ghostview yourfile.ps* and to get a hardcopy of the .ps file type *lpr harvgam0.ode.ps* which directs the .ps file to the printer.

2. THE SPECIFIC LAB PROJECT

1. **Constant Harvesting Model:** Using **xppaut** compute a dozen or so trajectories of $x(t)$ versus t for each of the three values $\gamma = 0$, $\gamma = .1$ and $\gamma = .4$. Explain analytically the different qualitative features observed and interpret the results biologically with regards to how the value of γ affects the possibility of extinction. *Turn in these plots to me and the analysis to support the numerical observations that there is a critical value of* γ.

2. **Periodic Harvesting Model:** A simple model where the harvesting is done on a periodic basis is

$$\frac{dx}{dt} = x(1-x) - (.15 + .15\sin(\gamma t)), \qquad x(0) = x_0$$

(Note this ODE is not autonomous and so our analytical techniques fail, However, we can still do numerical computations). Notice that the period of the harvesting is $2\pi/\gamma$. Neglecting the periodic term $.15\sin(\gamma t)$ we would have two equilibrium points since the constant level of harvesting would be less than the critical value of .25. The average value of the periodic term over a period is zero and there are two times on a given period where the harvesting is .30 and two times when there is no harvesting. The ode file for this case is *odes/harvestper.ode.* Please load this file into XPPAUT in the same way as for harvest.ode. Using **xppaut** compute some trajectories of $x(t)$ versus t for the value $\gamma = 4$ and for a few other values of γ. In particular, suppose that we take γ to very large (like $\gamma = 20$), what happens then? Comment on the main biological differences between constant harvesting and periodic harvesting? What happens as the period of the harvesting changes from small (i.e. from $\gamma = 4$ to $\gamma = 20$)? Are we more likely to achieve extinction? *Turn in plots as well as your written observations and conclusions.*