MATH 345 LAB 1 SOLUTIONS

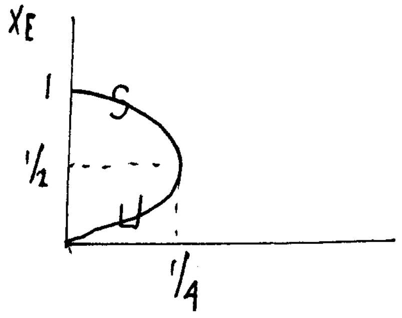
PROBLEM 1

THE HARVESTING MODEL IS

$$\frac{dx}{dt} = x(1-x) - \gamma$$

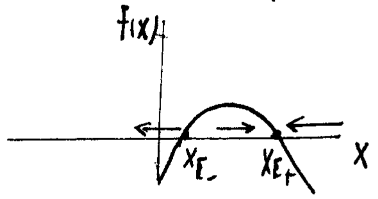
SO THAT x_E SATISFIES $x_E^2 - x_E + \gamma = 0$, $\quad x_{E_\pm} = \dfrac{1 \pm \sqrt{1-4\gamma}}{2}$.

THUS SADDLE-NODE BIFURCATION AT $\gamma = 1/4$.



(i) FOR $\gamma > 1/4$ NO EQUILIBRIA $\dot{x} < 0$ always

(ii) FOR $0 < \gamma < 1/4$

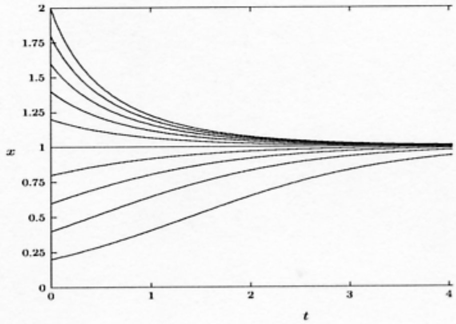


BIOLOGICAL INTERPRETATION.

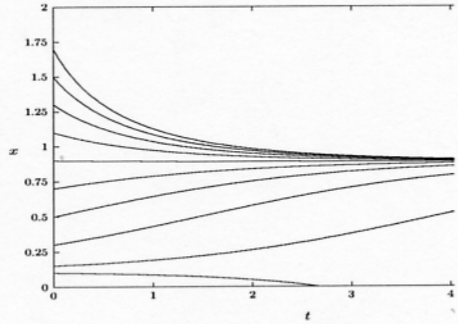
(i) IF NO HARVESTING $\gamma = 0$, THEN $x \to 1$ AS $t \to \infty$ SO THAT THE LIMITING STATE DEPENDS ON THE CARRYING CAPACITY OF THE SYSTEM.

(ii) IF $0 < \gamma < 1/4$, THEN $x(t) \to x_{E+}$ AS $t \to \infty$ WHEN $x(0) > x_{E+}$. HOWEVER, FOR ANY $x(0) < x_{E-}$ THEN $x \to x_{E+}$ AS $t \to \infty$ ALSO. BUT WHEN $x(0) < x_{E-}$, THEN $x(t) = 0$ AT SOME FINITE TIME T.

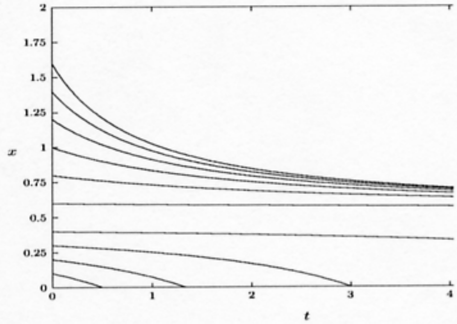
(iii) IF $\gamma > 1/4$ (TOO MUCH HARVESTING) THEN $x(t) = 0$ AT SOME FINITE TIME FOR ANY $x(0) > 0$. EXTINCTION ALWAYS OCCURS.



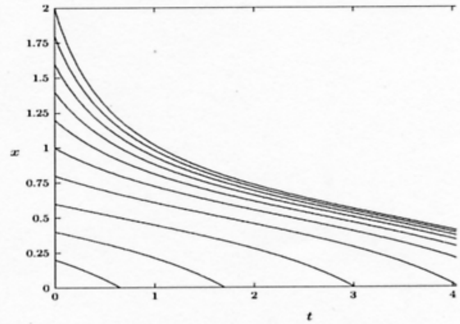
(a) $\gamma = 0$



(b) $\gamma = 0.1$



(c) $\gamma = 0.25$



(d) $\gamma = 0.40$

Figure 1: Plots of x versus t for different initial conditions and values of γ for $x' = x(1-x) - \gamma$.

PERIODIC HARVESTING

$$x' = x(1-x) - (0.15 + 0.15 \sin(\gamma t)).$$

ON THE NEXT PAGE WE HAVE PLOTTED TRAJECTORIES FOR

$\gamma = 1$, $\gamma = 4$, $\gamma = 10$, AND $\gamma = 20$.

WITHOUT HARVESTING PERIODICALLY IN TIME, THEN FOR

$x' = x(1-x) - 0.15$ WE GET $x(t) = 0$ AT $t = T < \infty$ WHEN

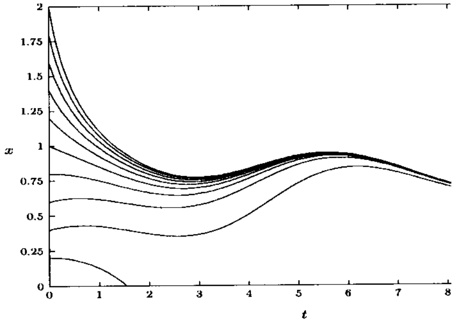
$$x(0) < \frac{1}{2} - \frac{1}{2}\sqrt{.40} \cong .183.. = x_{E}.$$

FROM THESE GRAPHS WE OBSERVE

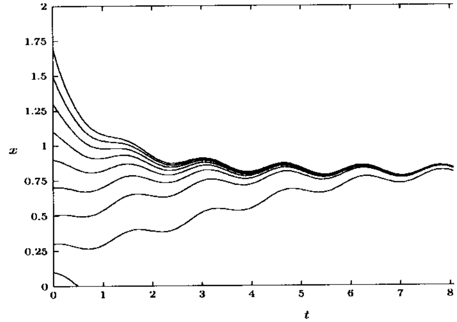
(i) THE POPULATION $x(t)$ IS ALSO PERIODICALLY VARYING AROUND SOME AVERAGED VALUE. THIS IS MORE PRONOUNCED AS γ INCREASES. THE FLUCTUATIONS BECOME OF HIGHER FREQUENCY AND SMALLER AMPLITUDE AS γ INCREASES.

(ii) ONE CAN CROSS BELOW $x(t) = .183 = x_E$ FOR SOME TIME INTERVAL WITHOUT GOING EXTINCT. THUS, THE POPULATION CAN START AT A SMALLER LEVEL WITH PERIODIC HARVESTING WITHOUT RISKING EXTINCTION.

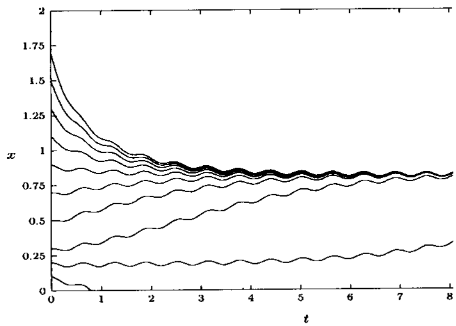
(iii) IT STILL APPEARS AS THOUGH $x(t) \to x_{E_+} = \frac{1}{2} + \frac{1}{2}\sqrt{.4} \simeq 0.89$ AS $t \to \infty$ (BUT NOT MONOTONICALLY).



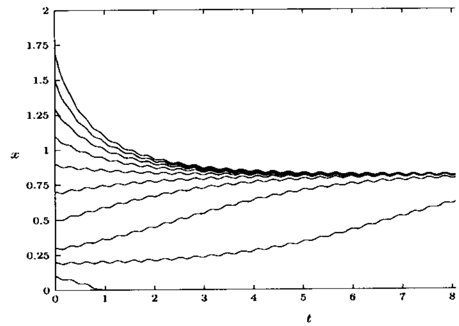
(a) $\gamma = 1$



(b) $\gamma = 4$



(c) $\gamma = 10$



(d) $\gamma = 20$

Figure 1: Plots of x versus t for different initial conditions and values of γ for $x' = x(1-x) - (0.15 + 0.15\sin(\gamma$