

Math 401: Homework 2

Instructions: Only turn in solutions to the questions marked in blue. However, you should look at the remaining questions

1. (10 points) For each of the problems below, find whether a solvability condition is needed for $f(x)$, and if so find this solvability condition. Assuming that this condition is satisfied, calculate the modified Green's function and find an integral representation for the solution $u(x)$ in terms of this Green's function.

(a) (5 points)

$$u'' = f(x), \quad 0 < x < 1; \quad u(0) = 0, \quad u'(1) = u(1).$$

(b) (5 points)

$$u'' + \pi^2 u = f(x), \quad 0 < x < 1; \quad u(0) = 0, \quad u(1) = 0.$$

(c) (0 points)

$$[(1 - x^2)u']' = f(x), \quad -1 < x < 1; \quad u \text{ is bounded at } x = \pm 1.$$

2. (10 points) Consider the following non-self adjoint boundary value problem with an inhomogeneous boundary condition at $x = 1$:

$$u'' + 2u' + u = f(x), \quad 0 < x < 1; \quad u(0) = 0, \quad u(1) = 1. \quad (1)$$

Find an integral representation for the solution $u(x)$ in two different (but equivalent) ways.

(a) (5 points) By first finding the Green's function for the adjoint operator L^* .

(b) (5 points) By first multiplying the equation in (1) by some function to make the resulting problem self-adjoint. Then, find the Green's function for this new problem and write the integral representation for u in terms of this Green's function.

3. (5 points) Let $\kappa > 0$ be arbitrary and consider the following non self-adjoint boundary value problem:

$$u'' + \kappa^2 u = f(x), \quad 0 < x < 1; \quad u(0) = u(1), \quad u'(0) = -u'(1).$$

Find a condition on $f(x)$ for this problem to have a solution. (Hint: you will need to find a nontrivial solution to the homogeneous adjoint problem after first deriving the adjoint boundary conditions).