MATH 556: Industrial Mathematics
Instructor: Michael Ward; Term 2 Jan-April 2026

Course Material and Topics:

In this course we formulate and analyze continuum-based mathematical models of phenomena in a wide range of areas of application. A list of the topics for the course is given below. The mathematical modeling typically leads to PDE/ODE models, which can be analyzed by dynamical systems and asymptotic techniques. For each topic covered I plan to give some insight into the formulation of appropriate mathematical models from the underlying physics. Wherever possible, asymptotic analysis is then used to systematically simplify the underlying model into a more tractable form. Asymptotic and numerical methods, based on MATLAB, will then be used to provide insight into the solution behavior. Finally, for many of the topics considered, we will examine a recent journal article (either in an Applied Math or Engineering journal) that is closely related to the topic. One of the main mathematical themes that links the different areas is stability and bifurcation theory.

Topics to be covered:

- Nonlinear Oscillators: Forced Oscillators, Entrainment, Stick-Slip Oscillations, Synchronization of Oscillators
- Floquet Theory: stability of periodic solutions
- Dynamical Hysteresis for ODE's; Slow passage problems
- Delay-Differential Equations; Machine-Tool Vibrations, Car-Following Models
- Bifurcation problems in combustion theory and nonlinear heat transfer
- Materials Science Modeling: MEMS modeling, theroelastic contact
- Lubrication theory and slow viscous flow phenomena; singularity formation

Prerequisites: This course is appropriate for theoretically minded engineering graduate students, Applied Math graduate students, and students interested in more theoretical issues in PDE/ODE. Students should have a working knowledge of Applied PDEs and ODEs (M400), and a little complex variables such as (M300,M301,or M305). A course in asymptotic analysis (M550) and some exposure to MATLAB is helpful **but not at all necessary.** Finally, previous exposure to continuum modeling in some area of application (such as fluid mechanics, materials science, or math biology, etc.) would be helpful.

Evaluation: The instructional format for the course will consist of lectures of 3 hours per week. There will be bi-weekly homework assignments. There will also be a final project where the students are to analyze a particular model appearing in a recent journal article and try to carefully go through some of the details of the paper, and to give a brief presentation of their paper. There is no final exam. The grading is 75% homework and 25% project.

Possible references: I will provide an online set of course notes for the various topics, as there is no one book that covers the core topics. Much of the course material is extracted from research articles. The following text is very useful for material on nonlinear oscillators:

- R. Rand; Lecture Notes in Nonlinear Vibrations, The Internet-First University Press (2012).