Math 100. Quiz 2. 2017-10-06 (Friday) **Time 25min**

Section ....   Instructor name ....

Your email ....

- **For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.**

- Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\frac{1}{100}$, $\sqrt{e}$ or $\ln(4)$ rather than decimals.

1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

   (a) **(1pt)** Find a **positive integer** n such that the equation

   $$x^3 - 3x = 4$$

   has a solution in the interval $[n, n+1]$.

   (b) **(1pt)** Compute the derivative of the function

   $$f(x) = \frac{3x - 2}{2x + 5}$$

   You must simplify your answer.

2. Each part of this question is worth 2 marks. **You have to show all your work in order to get credit.**

(a) **(2pt)** Use **the definition of continuity** to find all values a and b such that

$$f(x) = \begin{cases} \dfrac{x^2 + 2x + a}{x} & \text{for } x > 0, \\ b - 3x & \text{for } x \leq 0 \end{cases}$$

is **continuous everywhere**.

(b) **(2pt)** Find the x-coordinates of the points on the graph of $y = x^3 - 5x$ where the tangent line is parallel to the line $y = 70x + 1$.

3. This question is worth 4 marks. **You have to show all your work in order to get credit.**

Find all positive real numbers a with the property that the function

$$g(x) = \begin{cases} x^a \left(\cos(1/x) - 2\right) & \text{if } x > 0, \\ 0 & \text{if } x \leq 0 \end{cases}$$

is differentiable at $x = 0$. **Justify your answer using the definition of the derivative.**

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1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

   (a) **(1pt)** Find a **positive integer** n such that the equation

   $$x^3 - 1 = 5x$$

   has a solution in the interval $[n, n+1]$.

   (b) **(1pt)** Compute the derivative of the function

   $$f(x) = \frac{5x - 4}{3x + 2}$$

   You must simplify your answer.

2. Each part of this question is worth 2 marks. **You have to show all your work in order to get credit.**

(a) **(2pt)** Use **the definition of continuity** to find all values a and b such that

$$f(x) = \begin{cases} \dfrac{x^2 - a}{x - 1} & \text{for } x > 1, \\ b - 3x + x^2 & \text{for } x \leq 1 \end{cases}$$

is **continuous everywhere**.

(b) **(2pt)** Find the x-coordinates of the points on the graph of $y = x^3 + 3$ where the tangent line is parallel to the line $y = 48x + 48$.

3. This question is worth 4 marks. **You have to show all your work in order to get credit.**

Find all positive real numbers a with the property that the function

$$f(x) = \begin{cases} x^a \left(\sin(1/x) + 2\right) & \text{if } x > 0, \\ 0 & \text{if } x \leq 0 \end{cases}$$

is differentiable at $x = 0$. **Justify your answer using the definition of the derivative.**