Q2-T-p

Math 100. Quiz 2. 2017-10-05 Thursday. **Time 25min.**

Section .... Instructor name ....

Your email ....

- **For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.**

- Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\frac{1}{150}$, $\sqrt{e}$ or $\ln(4)$ rather than decimals.

1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

   (a) **(1pt)** For $f(x) = (1 + x^2)\sqrt{x}$, compute $f'(1)$.

   (b) **(1pt)** There is a car on a highway, whose location at time t is given by $y(t) = 80t + 30\cos t$. Find its instantaneous speed at $t = \pi/2$. (Ignore the units.)

2. Each part of this question is worth 2 marks. **You have to show all your work in order to get credit.**

(a) **(2pt)** Find the equation of the tangent line to the graph of $y = \sin x + e^x$ at $x = 0$.

(b) **(2pt)** Show that there is a real number x satisfying the equation

$$x^2 - 1 = \tan(x).$$

3. This question is worth 4 marks. **You have to show all your work in order to get credit.**

Use the definition of the derivative to find a and b such that the following function

$$f(x) = \begin{cases} x^5 + ax + b & \text{if } x \leq 0 \\ x^2 \sin\left(\frac{1}{x}\right) & \text{if } x > 0 \end{cases}$$

is differentiable at $x = 0$. You must justify your answer.

CŠŽFŽe

Math 100. Quiz 2. 2017-10-05 Thursday. **Time 25min.**

Section . . . .   Instructor name . . . .

Your email . . . .

- **For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.**

- Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\frac{1}{150}$, $\sqrt{e}$ or $\ln(4)$ rather than decimals.

1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

   (a) **(1pt)** For $f(x) = \dfrac{1}{1 + \sqrt{x}}$, compute $f'(1)$.

   (b) **(1pt)** There is a cyclist on 10th Avenue, whose location at time t is given by $y(t) = 15t - 5\sin t$. Find its instantaneous speed at $t = \pi$. (Ignore the units.)

2. Each part of this question is worth 2 marks. **You have to show all your work in order to get credit.**

(a) **(2pt)** Find the equation of the tangent line to the graph of $y = 2e^x + \cos x$ at $x = 0$.

(b) **(2pt)** Show that there is a real number x satisfying the equation

$$2x^2 = \tan x + 1.$$

3. This question is worth 4 marks. **You have to show all your work in order to get credit.**

Use the definition of the derivative to find a and b such that the following function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) + a & \text{if } x < 0 \\ 3x^2 + (2 + b)x & \text{if } x \geq 0. \end{cases}$$

is differentiable at $x = 0$. You must justify your answer.