Math 100. Quiz 5 2017-11-17 (Friday) **Time 25min**

Section ..... Instructor name .....

Your email .....

- **For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.**
- Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\frac{1}{100}$, $\sqrt{e}$ or $\ln(4)$ rather than decimals.

1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

(a) **(1pt)** Let $f(x) = x^4 + 3x^2 + 8$, and let $T_3(x)$ be its third-degree Taylor polynomial about $x = 1$. Evaluate $T_3''(1)$.

(b) **(1pt)** Find the smallest value for the parameter a such that the function

$$f(x) = (x + a)e^x$$

is increasing on the interval $(-1, \infty)$.

2. You have to show all your work in order to get credit.

(a) **(2pt)** Find the x-coordinates of the global minimum points for $f(x) = \frac{1}{\sqrt{x}} + \sqrt{x}$ on the interval $[\frac{1}{4}, 4]$.

(b) **(2pt)** Consider the function $f(t) = t^2 + \cos(t)$ defined for all real values t. Prove that it has at most one **critical point**.

3. You have to show all your work in order to get credit.
Let $f(x) = \ln(1 + 3x)$.

(a) **(1pt)** Use the 2nd degree Taylor polynomial to estimate $f(1/9)$.

(b) **(2pt)** Show that the error (in absolute value) of your estimate is smaller than 3^{-4}.

(c) **(1pt)** Determine whether your estimate is an overestimate or underestimate. You have to justify your answer.

Math 100. Quiz 5 2017-11-17 (Friday-p) **Time 25min**

Section ..... Instructor name .....

Your email .....

- **For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.**
- Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\frac{1}{100}$, $\sqrt{e}$ or $\ln(4)$ rather than decimals.

1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

(a) **(1pt)** Let $f(x) = x^4 - 4x^2 + x + 2$, and let $T_3(x)$ be its third-degree Taylor polynomial about $x = 1$. Evaluate $T_3''(1)$.

(b) **(1pt)** Find the largest value for the parameter a such that the function

$$f(x) = (x - a)e^{-x}$$

is decreasing on the interval $(-1, \infty)$.

2. You have to show all your work in order to get credit.

(a) **(2pt)** Find the x-coordinates of the global minimum points for $f(x) = \dfrac{2x}{1+x^2}$ on the interval $[-2, 2]$.

(b) **(2pt)** Consider the function $f(t) = \cos(t) - t^2 + 1$ defined for all real values t. Prove that it has at most one **critical point**.

3. You have to show all your work in order to get credit.
Let $f(x) = \ln(1 + 2x)$.

(a) **(1pt)** Use the 2nd degree Taylor polynomial to estimate $f(1/8)$.

(b) **(2pt)** Show that the error (in absolute value) of your estimate is smaller than $\dfrac{1}{3(2)^6}$.

(c) **(1pt)** Determine whether your estimate is an overestimate or underestimate. You have to justify your answer.