Math 100. Quiz 5. 2017-11-16 (Thursday) **Time 25min**

Section ..... Instructor name .....

Your email .....

- **For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.**
- Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\frac{1}{100}$, $\sqrt{e}$ or $\ln(4)$ rather than decimals.

1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

(a) Find the x-coordinates of the **local minimum** points of the function $f(x) = x^3 - 3x + 5$ defined on the whole real line.

(b) **(1pt)** Let $T_3(x)$ be the third degree Taylor polynomial about $x = 0$ of $g(x) = \frac{x}{1+x}$. Evaluate $T_3'(0)$.

2. You have to show all your work in order to get credit.

(a) **(2pt)** Find the x-coordinates of the **global maximum** points of $h(x) = x^5 - 5x + 5$ on $[0, 2]$.

(b) **(2pt)** Let $T_n(x)$ be the nth degree Taylor polynomial about $x = 0$ for the function $f(x) = \sin(x)$. Determine whether $T_{99}(0.1)$ gives an underestimate or overestimate of $\sin(0.1)$. Justify your answer.

3. You have to show all your work in order to get credit.
Let $\ell(x) = x^4 + 6x^2 + 4x + 2$.

(a) **(2pt)** Prove that $\ell(x)$ has at least one **critical point**.

(b) **(2pt)** Prove that $\ell(x)$ has at most one **critical point**.

Math 100. Quiz 5. 2017-11-16 (Thursday-p) **Time 25min**

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- **For each computation of limits in this test, if the limit does not exist, indicate whether it diverges to $-\infty$ or $+\infty$.**
- Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\frac{1}{100}$, $\sqrt{e}$ or $\ln(4)$ rather than decimals.

1. Each part of this question is worth 1 mark, and the correct answer will get the full mark.

(a) Find the x-coordinates of the **local maximum** points of the function $f(x) = x^3 - 12x - 1$ defined on the whole real line.

(b) **(1pt)** Let $T_3(x)$ be the third degree Taylor polynomial about $x = 1$ of $g(x) = x^2 e^x$. Evaluate $T_3'(1)$.

2. You have to show all your work in order to get credit.

(a) **(2pt)** Find the x-coordinates of the **global minimum** points of $h(x) = 3x^4 - 8x^3 + 6x^2 + 1$ on $[-1, 1]$.

(b) **(2pt)** Let $T_n(x)$ be the nth degree Taylor polynomial about $x = 0$ for the function $f(x) = \sin(x)$. Determine whether $T_{101}(0.1)$ gives an underestimate or overestimate of $\sin(0.1)$. Justify your answer.

3. You have to show all your work in order to get credit.
Let $\ell(x) = x^6 + 4x^2 + x + 2$.

(a) **(2pt)** Prove that $\ell(x)$ has at least one **critical point**.

(b) **(2pt)** Prove that $\ell(x)$ has at most one **critical point**.